

Journal Pre-proof

A model of thresholding bubble coalescence applied to underwater buoyancy-driven gas emission

Xin Ren, Jianghui Li, Tianyu Xiao, Daolong Zhou, Xuheng Yang, Fengling Yu, Xiaokang Zhang

PII: S0009-2509(26)01398-9
DOI: <https://doi.org/10.1016/j.ces.2026.124683>
Reference: CES 124683



To appear in: *Chemical Engineering Science*

Received date: 28 December 2025
Revised date: 13 July 2026
Accepted date: 14 July 2026

Please cite this article as: Xin Ren, Jianghui Li, Tianyu Xiao, Daolong Zhou, Xuheng Yang, Fengling Yu, Xiaokang Zhang, A model of thresholding bubble coalescence applied to underwater buoyancy-driven gas emission, *Chemical Engineering Science* (2026), doi: <https://doi.org/10.1016/j.ces.2026.124683>

This is a PDF of an article that has undergone enhancements after acceptance, such as the addition of a cover page and metadata, and formatting for readability. This version will undergo additional copyediting, typesetting and review before it is published in its final form. As such, this version is no longer the Accepted Manuscript, but it is not yet the definitive Version of Record; we are providing this early version to give early visibility of the article. Please note that Elsevier's sharing policy for the Published Journal Article applies to this version, see: <https://www.elsevier.com/about/policies-and-standards/sharing#4-published-journal-article>. Please also note that, during the production process, errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2026 Published by Elsevier Ltd.

Highlights

- A multi-stage screening mechanism is proposed for bubble coalescence.
- A new framework linking bubble size and velocity to coalescence.
- Model achieves 84.6% (single) and 81.5% (dual) accuracy in experimental validation.
- Bubble deformation and wake effects modulate coalescence processes.

Journal Pre-proof

A model of thresholding bubble coalescence applied to underwater buoyancy-driven gas emission

Xin Ren^a, Jianghui Li^{a,b,c,*}, Tianyu Xiao^a, Daolong Zhou^a, Xuheng Yang^d, Fengling Yu^{a,b} and Xiaokang Zhang^{a,c}

^aCollege of Ocean and Earth Sciences, Xiamen University, Xiamen, Fujian, China

^bState Key Laboratory of Marine Environmental Science, Xiamen University, Xiamen, Fujian, China

^cKey Laboratory of Underwater Acoustic Communication and Marine Information Technology Ministry of Education, Xiamen University, Xiamen, Fujian, China

^dSchool of Informatics, Xiamen University, Xiamen, Fujian, China

ARTICLE INFO

Keywords:

bubble coalescence
buoyancy-driven
gas emission
threshold model

ABSTRACT

Bubble coalescence plays a critical role in controlling the transmission, dissolution, and acoustic properties of underwater gas columns, which in turn directly impacts the quantification of gas flux. This study presents a theoretical model for bubble coalescence, suggesting that the process is a screening mechanism constrained by multiple physical interactions. We consider three main criteria: energy competition, hydrodynamic effects, and film drainage dynamics, and based on key bubble parameters (radius and velocity), we developed a coalescence threshold model under buoyancy-driven conditions. The model's accuracy was validated using single-nozzle and double-nozzle release experiments, with prediction accuracies of 84.6% and 81.5%, respectively. Furthermore, we address the limitations of the spherical assumption and steady-state flow in the theoretical model by exploring the impact of nonlinear behaviors such as bubble deformation and wake effects on the coalescence process. The study demonstrates that the proposed model effectively connects the bubble characteristic parameters with coalescence conditions and provides a unified theoretical framework for understanding bubble coalescence in complex multiphase flow systems.

1. Introduction

Bubble collision and coalescence phenomena are widespread in multiphase flow systems, playing critical roles in both industrial and environmental processes [1, 2]. These include chemical reactors [3], flotation devices [4, 5], nuclear reactor cooling systems [6, 7], and underwater gas leakage monitoring systems [8–10]. In marine environments, such interactions also govern the dynamics of seabed gas leakage, significantly influencing the fate of methane and carbon dioxide within the water column [11, 12]. It is estimated that global seabed methane emissions amount to approximately $(2\text{--}6) \times 10^7$ t annually, of which 5%–15% directly enters the atmosphere, intensifying the greenhouse effect [11]. Moreover, gas dissolution during bubble ascent may contribute to ocean acidification and the expansion of hypoxic zones, posing threats to benthic ecosystems [13–15].

Accurate quantification of these gas fluxes requires a reliable description of how bubble coalescence affects the bubble size distribution (BSD), as BSD governs both the acoustic and transport characteristics of bubble plumes [16]. However, many existing models fail to account for the complex coupling mechanisms between bubbles, leading to inaccuracies in BSD prediction and significant underestimation of flux [17, 18]. Existing models show two major shortcomings. First, they rely on the single bubble assumption and overlook the characteristics of group interactions. Second, mechanisms such as turbulence and buoyancy are often considered in isolation, without integrating the coupled effects

of membrane drainage dynamics and force balance. Although previous studies have examined these mechanisms, they remain incomplete. For example, Das [19] highlighted the probabilistic distribution of relative bubble velocity in turbulence but did not incorporate membrane drainage dynamics. Ozan et al. [20] proposed a coalescence kernel that coupled membrane drainage with critical velocity, yet their framework did not refine the force balance criterion for buoyancy-driven plumes. Federle et al. [21] demonstrated in flotation systems that coalescence depends not only on drainage and sliding times but also on the distribution of bubble density, spacing, and collision angle. Nevertheless, these insights have not been systematically applied to underwater gas emissions.

Bubble coalescence has been increasingly acknowledged as a dominant factor influencing both the evolution of BSD and the hydrodynamic behavior of bubble plumes [18, 22]. Both experimental and numerical studies demonstrate that when the volumetric gas fraction exceeds 10^{-4} , coalescence frequency increases sharply, substantially altering bubble plume behavior [23, 24]. However, field observations reveal that most bubble collision events do not lead to coalescence, instead resulting in non-contact movement or rebound after impact [23, 25]. Both Yang et al. [26] and Duineveld [27] investigate bubble coalescence behaviors, highlighting the crucial role of the Weber number in determining coalescence outcomes. They also emphasize that the collision eccentricity significantly influences coalescence behavior, with the impact varying based on the interface conditions. This indicates that bubble coalescence results from the synergistic

*E-mail address: jli@xmu.edu.cn (J. Li)

action of multiple timescales and mechanisms. First, bubbles must overcome flow resistance to achieve physical contact. Second, the liquid film between the two bubbles must drain within the available interaction time. If neither condition is satisfied, the collision leads to rebound or separation.

To understand bubble interactions in various flow regimes, previous studies have developed several bubble collision theories supported by both experimental observations and numerical simulations [23, 28]. These studies have identified a range of mechanisms that influence collision dynamics under various hydrodynamic conditions, including turbulence-induced pulsation, shear-induced deformation, wake entrainment, and buoyancy-driven overtaking [9, 23, 25, 29, 30]. Based on these findings, bubble collision mechanisms are classified into four categories: (i) Turbulence-induced collisions, where bubbles are treated as inertial tracers and the collision frequency is governed by local turbulent energy dissipation—though such models struggle under buoyancy-dominated and turbulence-heterogeneous conditions [25]; (ii) Shear-induced collisions, which account for velocity gradients but neglect wake effects and bubble deformation under buoyant forces [29, 31]; (iii) Buoyancy-induced collisions, based on differences in terminal velocities, but often fail to incorporate local hydrodynamic interactions or transient flow behavior [9]; (iv) Wake-induced entrainment collisions, where trailing bubbles are drawn into the low-pressure wake of leading bubbles—although their predictive capacity is limited outside the near-wake region [30].

These models typically address individual mechanisms in isolation and fail to capture their combined effects, which limits predictive accuracy in buoyancy-driven plumes [23, 32]. Therefore, a multi-criteria threshold model that integrates membrane drainage dynamics, energy competition, and force balance is required to improve the reliability of bubble coalescence prediction in complex flow fields and to provide a robust theoretical basis for gas flux quantification.

This paper is organized as follows. Section 2 presents the theoretical framework for bubble coalescence. Section 3 describes the data collection and processing procedures. Section 4 reports the experimental results and mechanistic analysis. In Section 5, conclusions and discussion are summarized.

2. Physical mechanism and coalescence criterion

The bubble coalescence process can typically be divided into three distinct stages: (i) an initial bubble pair, (ii) contacting but unmerged bubbles, and (iii) a single merged bubble. These state transitions are governed by two key dynamic processes: the approach stage and the contact stage, each dominated by different mechanisms that collectively determine whether coalescence occurs (Fig. 1).

In the approach phase, bubbles rise under buoyant forces and may approach each other because of hydrodynamic interactions. Whether contact is achieved depends on the net

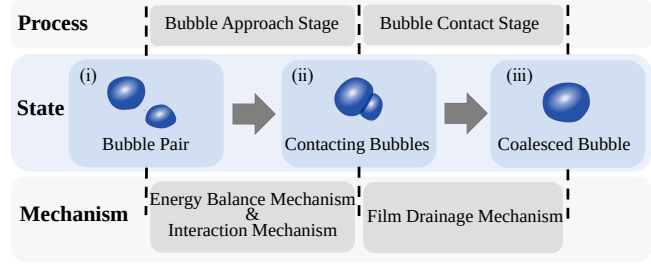


Fig. 1: Schematic of the three-stage bubble coalescence process: (i) an initial bubble pair, (ii) contacting but unmerged bubbles, and (iii) a single merged bubble. The transitions are governed by approach and contact dynamics.

driving force relative to viscous and inertial resistance, as well as the presence of fluid-induced attraction.

In the contact stage, even though bubbles come into surface contact, a thin liquid film remains between them. Coalescence only occurs if this film is sufficiently drained and ruptured within the contact duration. Therefore, the duration of inter-bubble contact becomes a critical factor governing the final merging.

To quantitatively evaluate coalescence, we establish a criterion framework rooted in three physical mechanisms, each describing a distinct condition necessary for bubble merging:

- The mechanism of energy balance evaluates whether the bubble pair possesses sufficient kinetic energy to establish contact.
- The mechanism of interaction forces checks for attractive forces between the bubbles.
- The mechanism of the film's drainage focuses on whether the contact duration exceeds the complete drainage time.

2.1. Energy Based Criterion

2.1.1. Energy Composition

The bubble coalescence process is fundamentally a dynamic evolution driven by energy and constrained by dissipative mechanisms. It typically consists of two stages: (i) the inertial contact stage, governed by macroscopic motion, and (ii) the film drainage stage, dominated by lubrication effects and interfacial forces. This subsection focuses on the inertial contact stage, which encompasses the process from when bubbles approach each other at an initial velocity until their surfaces make initial contact. Coalescence between interacting bubbles occurs only when the net driving force exceeds the interfacial resistance. This threshold is determined by the energy balance at the critical separation distance d_{eq} [33, 34]:

$$W_{\text{driving}}(d_{eq}) = W_{\text{resisting}}(d_{eq}), \quad (1)$$

where W_{driving} is the driving forces that facilitate coalescence, and $W_{\text{resisting}}$ refers to the resisting forces that hinder it. During this stage, the system's energy conversion follows the conservation principle: the initial kinetic energy of the bubbles is used to overcome both viscous dissipation and

Buoyancy-driven bubble coalescence model

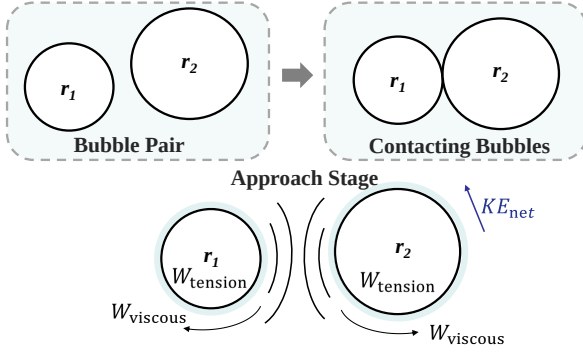


Fig. 2: Energy components in bubble approach stage. The schematic shows two bubbles under mutual attraction. Driving forces include net kinetic energy (KE_{net}). Resistance arises from viscous dissipation (W_{viscous}) and surface tension work (W_{tension}). Coalescence occurs when driving contributions overcome resistance.

the potential energy barrier caused by surface tension. If the kinetic energy is insufficient to overcome these dissipative forces, the bubbles will fail to make contact and will instead rebound. This principle is quantitatively represented by the energy balance (Fig. 2), which schematically outlines the energy pathways involved in the coalescence process.

$$KE_{\text{net}} = W_{\text{viscous}} + W_{\text{tension}} \text{ at } (d = d_{\text{eq}}), \quad (2)$$

where d denotes the instantaneous distance between the surfaces of two bubbles. As the bubbles approach each other, this distance varies with time and is reevaluated at each time frame. In the present model, d is taken as the instantaneous value at a specific instant immediately prior to contact.

This energy criterion defines the critical distance d_{eq} at which the cumulative driving energy exactly offsets the resisting forces. Physically, $d = d_{\text{eq}}$ represents the maximum separation required for energetically feasible coalescence.

This criterion captures the balance between dissipative energy mechanisms, such as viscous losses, and the interfacial rearrangement processes that occur during coalescence [33]. The principal energy terms contributing to this balance are discussed in the following paragraphs.

Kinetic energy: The initial kinetic energy KE_{net} arises from the relative motion of two interacting bubbles with radii r_1 and r_2 , approaching each other with a relative velocity v_{rel} . In the center-of-mass frame, the kinetic energy associated with the relative motion can be written as:

$$KE_{\text{net}} = \frac{1}{2} m_{\text{eff}} v_{\text{rel}}^2, \quad (3)$$

where m_{eff} denotes the effective (reduced) inertial mass of the interacting system, and $v_{\text{rel}} = |\vec{v}_1 - \vec{v}_2|$ is the relative velocity between the two bubbles. Here, $\vec{v}_1$ and $\vec{v}_2$ represent the velocity vectors of bubble 1 and bubble 2, respectively.

For gas bubbles rising in a liquid, the inertia is dominated by the displaced liquid rather than the gas mass. During the relative approach, a surrounding liquid region with a characteristic length scale comparable to $(r_1 + r_2)$ is accelerated. Accordingly, the effective inertial mass is estimated at the scaling

$$m_{\text{eff}} \approx \rho(r_1 + r_2)^3, \quad (4)$$

which represents the characteristic liquid volume involved in the interaction.

The kinetic energy is finally expressed as

$$KE_{\text{net}} = \frac{1}{2} k_1 \rho(r_1 + r_2)^3 v_{\text{rel}}^2, \quad (5)$$

where k_1 is a dimensionless inertial coefficient accounting for the effective volume of the surrounding liquid accelerated during the relative motion of the bubble pair.

Viscous dissipation work: The relative motion of bubbles in a viscous liquid leads to energy dissipation. In the present inertial approach stage, the liquid film between the bubbles has not yet formed, and the dominant dissipation mechanism arises from bulk (internal) viscous resistance in the surrounding liquid. Lubrication dissipation associated with thin film drainage is therefore not considered at this stage and will be discussed separately for the post-contact regime.

For a clean spherical bubble moving with velocity $|\vec{v}|$ in a quiescent liquid, Levich [28] derived the viscous resistance based on global energy dissipation balance as:

$$F_D = 12\pi\mu r |\vec{v}|, \quad (6)$$

where r is the bubble radius and μ is the dynamic viscosity of the liquid.

Extending this result to the relative approach of two bubbles along their line of centers, the bulk viscous dissipation during the inertial approach stage can be estimated by adopting the relative velocity v_{rel} as the characteristic velocity scale and the interaction distance d as the characteristic length scale. Accordingly, the viscous dissipation work is written as:

$$W_{\text{viscous}} = 6\pi\mu(r_1 + r_2) v_{\text{rel}} \cdot d, \quad (7)$$

This expression represents a scale estimate of bulk viscous dissipation during the inertial approach stage, rather than lubrication-dominated dissipation within a thin liquid film.

Surface tension resistance: To bring the surfaces of two bubbles into contact, an energy barrier must be overcome, primarily due to the local suppression and deformation of the gas-liquid interface before coalescence. When the two bubbles approach each other, the energy required to overcome this barrier is determined by the interfacial energy between the bubbles and the liquid.

Based on the interfacial energy theory of Israelachvili [35], the work required to overcome surface tension for the surfaces of two unit areas of bubbles is proportional to the interfacial energy. For the bubble-liquid interface, the interfacial energy is the gas-water interfacial tension σ [35].

For two approaching bubbles with radii r_1 and r_2 , the effective interaction region associated with interfacial deformation is governed by the local curvatures of the bubble surfaces near their closest approach. Under the experimental conditions considered in this study, the size difference between interacting bubbles is small. Therefore, the characteristic length scale of this interaction can be reasonably approximated as being proportional to the combined bubble

size $(r_1 + r_2)$. The geometric deviation arising from unequal curvatures is incorporated into a dimensionless geometric coefficient, and the implications of this simplification are discussed later. Combining this geometric scaling with the interfacial energy framework, the surface tension resistance work W_{tension} can be expressed as [36]:

$$W_{\text{tension}} = k_2 \cdot \sigma \cdot (r_1 + r_2), \quad (8)$$

where k_2 represents an effective geometric length scale that accounts for the characteristic interaction area during initial bubble contact, including the effects of local interface deformation, curvature geometry, and numerical prefactors introduced by the equal-size bubble approximation, and σ denotes the gas–water interfacial tension.

2.1.2. Distance Threshold Expression

The critical separation distance is obtained by rearranging the resulting equation as:

$$d_{\text{eq}} = \frac{k_1 \rho (r_1 + r_2)^2 v_{\text{rel}}^2 - 2k_2 \sigma}{12\pi \mu v_{\text{rel}}}. \quad (9)$$

This relationship indicates that the critical distance increases with the square of the relative velocity and the combined radius of the bubbles, representing a balance between inertial and interfacial forces.

2.2. Interaction Based Criterion

2.2.1. Attraction Mechanism

The dynamics of a rising bubble in a liquid are governed by Newton's second law, which relates the net external force acting on the bubble to the time rate of change of its linear momentum:

$$m \frac{dv}{dt} = \sum F. \quad (10)$$

In bubble dynamics, Newton's second law captures the combined influence of body forces and hydrodynamic interactions. When a single bubble reaches a quasi-steady rising state, the net force acting on it approaches zero. The corresponding force balance under this condition can be expressed as [37–39]:

$$m \frac{dv}{dt} = F_B + F_D + F_L + F_{VM} + F_G. \quad (11)$$

Each force term in Eq. (11) represents a distinct physical mechanism, as illustrated in (Fig. 3 (a)): (i) F_B , buoyancy due to density difference; (ii) F_D , drag from viscous resistance; (iii) F_L , lift induced by flow asymmetry; (iv) F_{VM} , virtual mass from fluid acceleration; (v) F_G , gravitational force from bubble mass.

Bubble interactions are governed by the direction and magnitude of the forces acting between them [40]. To facilitate analysis, complex bubble group systems are reduced to isolated bubble pairs, as illustrated in Fig. 3(b). Based on the interaction orientation, the resulting hydrodynamic force is decomposed into two components: a relative rotational force and a relative attractive force [41, 42].

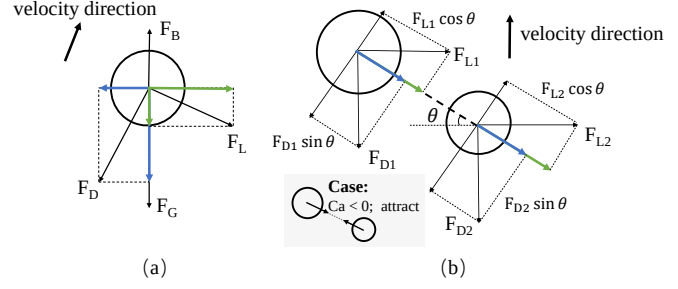


Fig. 3: Schematic illustration of forces acting on bubbles: (a) a single bubble; (b) a pair of interacting bubbles.

The relative attraction force F_a , which governs the net attractive or repulsive interaction between bubbles, is calculated by multiplying the dimensionless attraction coefficient C_a by the dynamic pressure term $\frac{\pi r_{ij}^2 \rho |\vec{v}|^2}{2}$ [42].

We adopt the equivalent-radius approximation, $r_{ij} = \frac{2r_1 r_2}{r_1 + r_2}$, to describe the hydrodynamic attraction between two interacting bubbles. Since the size difference between the bubbles in our experiments is only a few percent, this approximation can be considered a reasonable extension of classical equal-radius bubble interaction models. According to Kok's theoretical formulation, the attraction coefficient C_a is expressed as [42, 43]:

$$C_a = 2M^* - 2N^* \cos(2\theta), \quad (12)$$

where M^* and N^* are interaction parameters, expressed as functions of the normalized separation $S = \frac{d+2r_{ij}}{r_{ij}}$ [42, 43]:

$$\begin{cases} M^* = \frac{3S^{-4} - 15S^{-7} - 64S^{-9}}{2}, \\ N^* = \frac{9S^{-4} - 9S^{-7} - 32S^{-9}}{2}. \end{cases} \quad (13)$$

However, it is important to note that as the size difference between the bubbles increases, the asymmetry of the wake generated by the leading bubble may lead to deviations from the predictions of the equal-radius model. Therefore, in the discussion section, we assess the applicability of this approximation and quantify the potential errors, drawing upon existing research and experimental observations. This allows us to define the conditions under which the model remains valid and to suggest possible directions for future improvements.

2.2.2. Attraction Angle Threshold

Within the experimentally relevant range of dimensionless separation $1 < S \leq 5$, leading-order terms dominate the behavior of the attraction coefficient, allowing the omission of higher-order contributions [42]. Under this approximation, the expressions for M^* and N^* in Eq. 13 can be simplified as:

$$M^* \approx \frac{3S^{-4} - 15S^{-7}}{2}, \quad N^* \approx \frac{9S^{-4} - 9S^{-7}}{2}. \quad (14)$$

Substituting the above into Eq. 12 yields a linearized form of the attraction coefficient C_a :

Table 1

Critical attraction angles at different dimensionless separations

Dimensionless Separation S	Critical Angle θ_{crit}
2.0	40.9°
4.0	35.9°
6.0	35.5°
8.0	35.3°
10.0	35.3°

$$C_a = \frac{3}{S^4} (1 - 3 \cos 2\theta) + \frac{3}{S^7} (3 \cos 2\theta - 5). \quad (15)$$

In this formulation, $C_a < 0$ indicates the presence of net hydrodynamic attraction between the bubbles. To systematically investigate the angle dependence of the attraction effect at different spacings, we solved the inequality $C_a < 0$ to determine the critical angle θ . Table 1 lists the critical angle values corresponding to typical dimensionless spacings S .

When $S \leq 3.0$, the contribution of higher-order terms to the critical angle cannot be ignored. Taking into account the initial bubble release positions, release frequency, and the subsequent dynamical evolution of inter-bubble spacing observed in the experiments, the effective separation ratio during bubble interaction is considered to lie within the range $S = 2 - 10$ ($S = 2$ corresponds to the onset of bubble contact, while $S = 3$ represents a configuration with an inter-bubble gap on the order of one bubble radius [42]). Solving the condition $C_a < 0$ over this range yields a corresponding critical collision angle of approximately 35.3°–40.9°.

2.3. Drainage Time Criterion

2.3.1. Film Drainage Mechanism

Only a subset of colliding bubbles proceed to coalescence, while most rebound without merging [18]. To determine whether coalescence follows a collision, it is essential to incorporate collision duration as a key parameter in coalescence modeling frameworks.

According to the film-drainage model, bubble contact initiates the formation of a thin liquid film between the interfaces, which must drain to a critical thickness before coalescence can occur [44, 45]. If the drainage process is incomplete within the available contact duration, the bubbles will separate without merging [44]. During this contact stage, the interacting bubbles remain mechanically distinct, and coalescence is postponed until the intervening liquid film sufficiently thins and ruptures. A schematic illustration of the interfacial configuration and force balance characterizing this contact stage is provided in Fig. 4.

The predicted coalescence time is determined using the following formulation [44, 45]:

$$t_{ij} = \left(\frac{r_{ij}^3 \rho}{16\sigma} \right)^{1/2} \ln \frac{h_o}{h_f}, \quad (16)$$

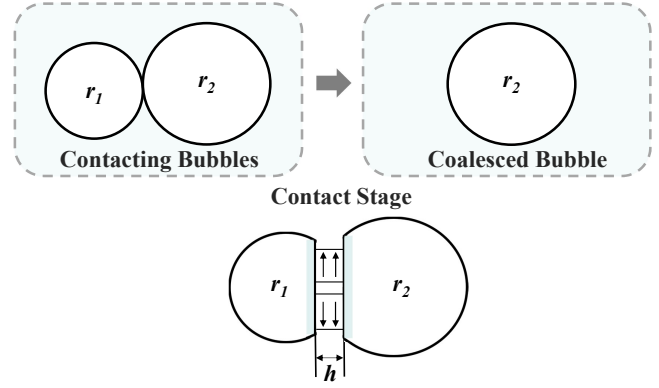


Fig. 4: Schematic illustration of the bubble contact stage: interacting bubble pair during the contact stage, where a thin liquid film forms and drains before coalescence.

where h_o and h_f represent the initial and critical thicknesses of the liquid film, respectively. Eq. 16 defines the characteristic drainage time t_{ij} , which determines whether the interfacial film can thin sufficiently before the bubbles separate. If the observed contact duration exceeds t_{ij} , the film has sufficient time to rupture, resulting in coalescence. Otherwise, shorter contact durations cause rebound, preventing coalescence.

2.3.2. Drainage Time Expression

The original nonlinear formulation for the minimum drainage time t_{ij} , given in Eq. 16 [44, 45], describes the time required for the intervening liquid film between two colliding bubbles to drain sufficiently for coalescence to occur.

Experimental and theoretical studies have shown that, in turbulent multiphase flows, although the flow conditions are complex, the variations in the initial liquid film thickness h_o and the critical rupture thickness h_f are relatively stable. Based on this, many classic studies treat $\ln \frac{h_o}{h_f}$ as a constant. For example, Prince [25] set $h_i = 10^{-4}$ m and $h_f = 10^{-8}$ m, thereby simplifying the logarithmic term as a constant. Similarly, Coulaloglou [45] adopted a similar assumption to simplify the drainage time expression. This approach is convenient for dimensional analysis and parameter estimation [46].

Based on this simplification, the drainage time can be further linearized as a power-law function of the bubble equivalent radius:

$$t_{ij} = k_3 r_{ij}^{3/2}, \quad (17)$$

where k_3 is a constant determined by the fluid properties and interfacial conditions.

2.4. Mechanism based coalescence criterion

To enable quantitative prediction of coalescence events, the three physical mechanisms described above are consolidated into a unified parameter-based criterion model.

Each condition is expressed in terms of experimentally measurable quantities, including bubble radius, relative velocity, separation distance, and contact duration. The resulting parametrised framework includes the following three conditions:

- Energy Based Distance Criterion: $d < d_{eq}$, where d_{eq} is determined by bubble size and velocity difference;
- Interaction Based Criterion: $C_a < 0$, where C_a depends on the collision angle and normalized separation;
- Drainage Time Based Criterion: $t > t_{ij}$, where t_{ij} is governed by bubble radius and interfacial film dynamics.

Where all instantaneous quantities (such as d , θ , v_{rel}) are time dependent and are reevaluated at each time frame throughout the approach. For the coalescence criterion, these quantities are consistently evaluated at the same instant immediately prior to contact. The complete coalescence criterion model is summarized as:

$$\begin{cases} d < d_{eq} = \frac{k_1 \rho (r_1 + r_2)^2 v_{rel}^2 - 2k_2 \sigma}{12\pi \mu v_{rel}}, \\ 35.3^\circ < \theta < 40.9^\circ \quad \text{at } (2 < S < 10), \\ t > t_{ij} = k_3 r_{ij}^{3/2}. \end{cases} \quad (18)$$

From a physical standpoint, the three conditions constrain the coalescence process from complementary perspectives, namely energy accessibility, dynamical approach feasibility, and the drainage time. These conditions are neither independent nor equivalent constraints; rather, each corresponds to a distinct physical stage of the coalescence process. Together, they form a sequential set of requirements that must be satisfied in order for the system to advance from one stage to the next.

This parametrized framework bridges the gap between physical theory and experimental observables. By transforming abstract mechanisms into explicit, testable conditions, it enables quantitative classification of coalescence outcomes and facilitates sensitivity analysis regarding key factors such as bubble size, velocity difference, and spatial orientation.

3. Data collection and processing

This section presents the experimental setup and implementation plan for investigating bubble coalescence. The YOLO V8-SORT framework [47–49] is employed to process the collected optical data, enabling the reconstruction of three-dimensional bubble trajectories and the simultaneous quantitative analysis of bubble parameters.

3.1. Experimental setup

The experiment was conducted in a transparent plexiglass water tank with dimensions of $30 \times 58 \times 35$ cm (Fig. 5). Gas was supplied by an air compressor, and bubbles were periodically formed at the nozzle tip. Two experimental configurations were adopted: a single-nozzle setup

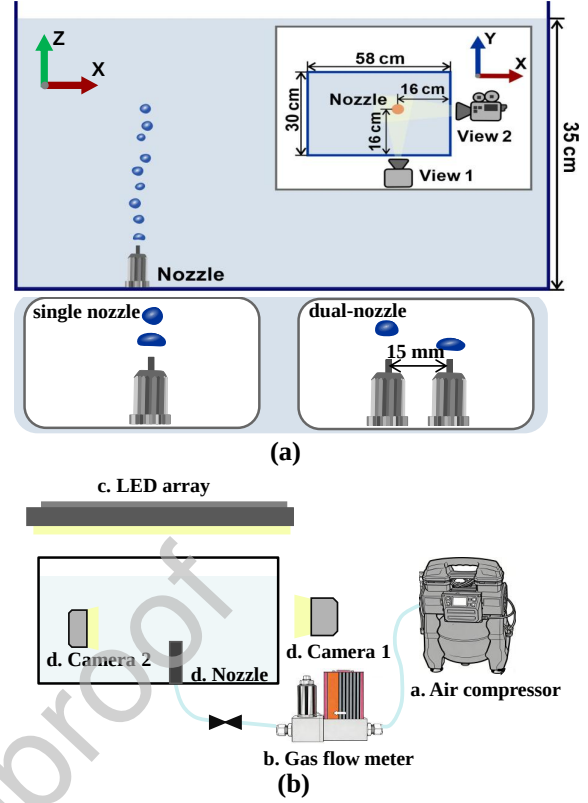


Fig. 5: Experimental schematic diagram. (a) Schematic of the water tank setup; (b) schematic of the experimental setup including compressor, flowmeter, nozzle and optical equipment.

generating axially aligned bubble pairs, and a dual-nozzle setup producing parallel bubble pairs [38, 50], as shown in Fig. 5. For each configuration, variations in nozzle diameters and flow rates were systematically controlled. The detailed equipment and control parameters are summarized in Table 2.

To capture the spatial evolution of the rising bubbles, two high-speed cameras were placed at right angles. One camera captured the motion along the XZ plane, and the other along the YZ plane. Each camera recorded at 1920×1080 resolution and 240 frames per second, enabling precise tracking of bubble trajectories. A checkerboard calibration target is used to align the two camera views geometrically. Coalescence phenomena of axially aligned and parallel-rising bubble pairs are shown in Fig. 6.

3.2. Data processing

Based on the optical data, we designed a workflow for extracting bubble parameters. The workflow consists of three stages: (i) preprocessing; (ii) detection and tracking; and (iii) parameter extraction. Fig. 7 shows the overall data processing framework.

The dual-view optical data undergo detailed preprocessing to ensure accurate subsequent analysis. First, the timestamps of the dual-camera data streams are aligned to ensure proper frame pairing. Then, geometric calibration

Buoyancy-driven bubble coalescence model

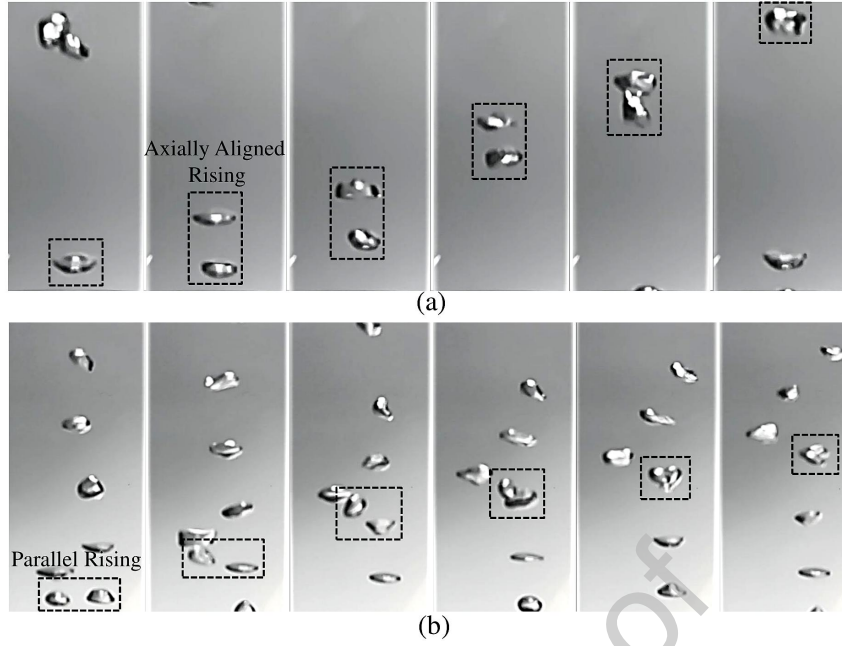


Fig. 6: Bubble coalescence dynamics under different release configurations. (a) Single-nozzle release of axially aligned rising bubble pairs; (b) dual-nozzle release of parallel rising bubble pairs.

Table 2

Experimental setup and controlled parameters

(a) Bubble release experimental conditions

Variable	Settings
Nozzle Type	Single-nozzle (linear release) Dual-nozzle (parallel release)
Inner diameter (mm)	2.3, 1.05, 0.86, 0.72, 0.6
Flow Rate (mL/min)	20, 40, 60, 80, 100

(b) Specifications of experimental equipment

Equipments	Parameters
(a) Air Source	Power: 1200W, Adjustable Output Pressure: 0.3-0.9 MPa (Stable at 0.3 MPa during experiments)
(b) Flowmeter	Range: 2-100 mL/min, Accuracy: $\pm 0.5\%$ FS
(c) Light Source	Type: White LED, Power: 900W
(d) Imaging Window	Size: 120×200 mm, Resolution: 1080×1920 pixels

is performed using a checkerboard target to correct lens distortion and establish a mapping between pixel coordinates and physical space.

The next stage involves reconstructing the three-dimensional trajectories of the rising bubbles using dual-view optical data. This process encompasses bubble detection, trajectory generation, and stereoscopic 3D reconstruction from

both camera views. The specific steps are as follows (Appendix 19):

Step 1: Bubble detection [47]. The YOLOv8 model detects bubble positions in each frame. For a bubble detected in view k (where $k = 1, 2$), the model outputs a bounding box defined by the coordinates of its top-left corner (x_k, y_k) and its pixel dimensions (w_k, h_k) . These pixel-based measurements are converted to physical lengths using the corresponding calibration factor α_k .

Step 2: Trajectory generation [48, 49]. The SORT algorithm integrates Kalman filtering for motion prediction and applies the Hungarian algorithm to associate detections across frames, thereby generating continuous trajectories.

Step 3: Dual-view 3D reconstruction. Temporal and spatial consistency between the two camera views is manually verified to ensure correct bubble matching. The three-dimensional centroid coordinates of each bubble are then calculated by fusing information from both orthogonal views. The x and y coordinates are obtained directly from the horizontal axes of Camera 1 and Camera 2, respectively. The depth coordinate (z) is computed as the average of the vertical positions from both views, providing a robust estimate that mitigates single-view parallax errors:

$$\begin{cases} C_x = \alpha_1 \cdot \left(x_1 + \frac{w_1}{2}\right), \\ C_y = \alpha_2 \cdot \left(x_2 + \frac{w_2}{2}\right), \\ C_z = \frac{\alpha_1 \cdot \left(y_1 + \frac{h_1}{2}\right) + \alpha_2 \cdot \left(y_2 + \frac{h_2}{2}\right)}{2}. \end{cases} \quad (19)$$

Based on the detection results and trajectory data, the following parameters are extracted to characterize bubble interaction behavior:

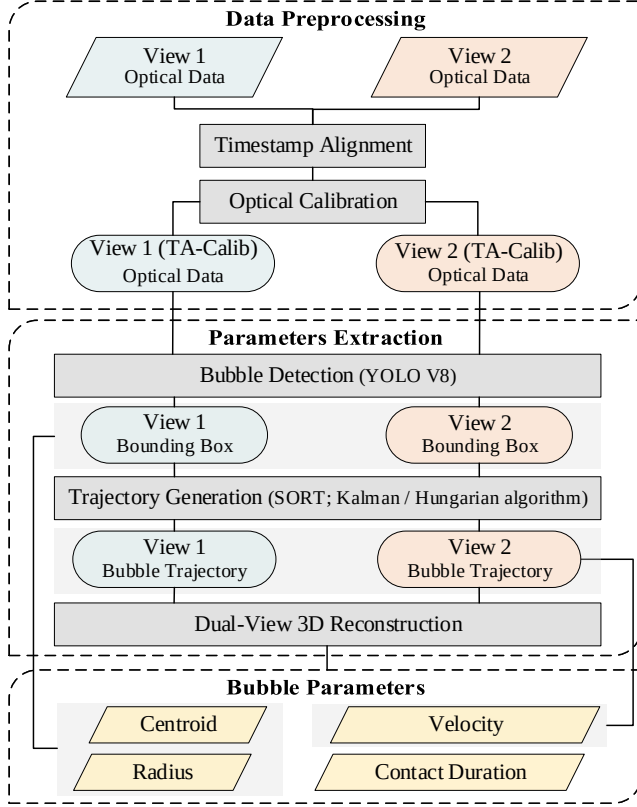


Fig. 7: Data preprocessing and bubble parameter extraction.

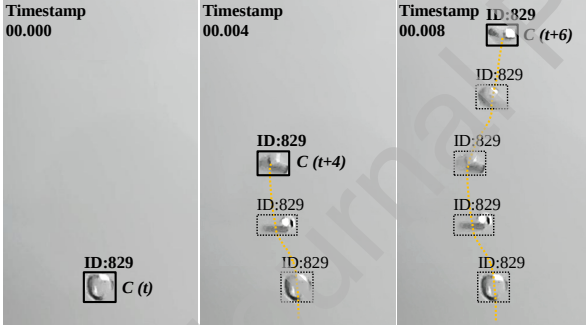


Fig. 8: Bubble trajectory tracking with YOLO detection.

- **Centroid ($C(t)$):** The time-varying geometric center of the bubble. While slight deviations between the geometric center and the center of mass may occur due to shape asymmetry, the bounding-box center calculated via Eq. (19) provides a consistent and robust representation of the bubble's trajectory [47].
- **Radius (r):** The equivalent radius of the bubble, derived from the bounding box dimensions and the calibrated pixel to length scaling. For non-spherical and wobbling bubbles, a unique geometric radius or well-defined principal axes are generally not available. In this study, bubble size is therefore characterized using a volume equivalent radius, defined as the radius of a sphere having the same estimated volume as the bubble, which

serves as a modeling scale rather than a strict geometric descriptor [28, 35]. For each bubble, if the three principal dimensions length $l = \alpha_1 w_1$, width $w = \alpha_2 w_2$, and height $h = (\alpha_1 h_1 + \alpha_2 h_2)/2$ are available, the volume equivalent radius is calculated as:

$$r = \sqrt[3]{\frac{l \cdot w \cdot h}{8}}. \quad (20)$$

- **Bubble surface distance (d):** The shortest distance between the surfaces of two bubbles. It is calculated by subtracting the sum of the equivalent radii of the two bubbles from the three-dimensional centroid distance:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} - (r_1 + r_2), \quad (21)$$

where $\Delta x = C_{x1} - C_{x2}$, $\Delta y = C_{y1} - C_{y2}$, $\Delta z = C_{z1} - C_{z2}$.

- **Velocity ($\vec{v}$):** The rising velocity of each bubble is calculated by differentiating its centroid positions over time using the SORT-based trajectories [48, 49]:

$$\vec{v}_t = \frac{\mathbf{C}(t + \Delta t) - \mathbf{C}(t)}{\Delta t}, \quad (22)$$

where $\mathbf{C}(t) = (C_x(t), C_y(t), C_z(t))$, and Δt is the time interval between consecutive frames.

- **Collision Angle (θ):** Defined as the angle between the vector connecting the centroids of the two bubbles and the vertical upward direction (the z-axis), which characterizes the spatial relative orientation of the bubble pair.

The centroid-connecting vector is defined as:

$$\vec{d}_c = (x_2 - x_1, y_2 - y_1, z_2 - z_1), \quad (23)$$

and the collision angle is calculated as [42, 43]:

$$\cos(\theta) = \frac{\vec{d}_c \cdot \vec{e}_z}{|\vec{d}_c|}, \quad (24)$$

where $\vec{e}_z = (0, 0, 1)$ is the unit vector in the vertical upward direction.

- **Contact Duration (t):** The contact time of bubbles is determined through manual frame-by-frame inspection of synchronized dual-view images. The contact start is defined as the frame at which no visible gap exists between the edge pixels of the two bubbles. The contact end is defined based on the collision outcome: for separation events, it corresponds to the frame at which a discernible gap reappears at the contact interface; for coalescence events, it corresponds to the frame at which the liquid film between the two bubbles ruptures, the interface contours merge into a single continuous structure, and the gas cores become connected.

Based on the resolution of the optical equipment and the imaging window, the spatial resolution of the experiment is approximately 0.11 mm per pixel. The time resolution is determined by the frame rate of the optical equipment, which is 240 fps (i.e., ± 4.17 ms). The measurement uncertainty for the contact time is approximately ± 1 frame. The systematic

Table 3

Time-resolved parameter dataset of a single bubble trajectory

Timestamp(s)	Geometric parameters (mm)				Motion parameters (m/s)			
	Centroid X	Centroid Y	Centroid Z	Radius	Velocity	VX	VY	VZ
00.000	0.00	0.00	0.00	1.95	0.00	0.00	0.00	0.00
00.004	-0.15	-0.02	1.05	1.97	0.26	-0.04	0.00	0.25
00.008	-0.30	-0.04	2.11	1.95	0.26	-0.03	0.02	0.25
00.012	-0.37	0.11	3.17	1.89	0.26	-0.02	0.04	0.26
00.017	-0.45	0.27	4.24	1.80	0.28	-0.02	0.02	0.28

error for extracting geometric parameters such as the radius using the YOLO bounding box is estimated to be around 3%.

Fig. 8 illustrates the trajectory generation of bubbles over time, along with the bounding boxes extracted by the YOLO object detection algorithm. Table 3 presents an example of the time-resolved bubble dataset.

3.3. Bubble interaction behaviors

Three distinct types of bubble interaction behaviors were identified during the experiments, each governed by a different physical process:

- **Non-contact:** Bubbles maintain a stable separation and rise synchronously in a spiral path, indicating that the relative distance is too large or there is insufficient attraction, preventing physical contact;
- **Bounce off:** Bubbles briefly form a thin liquid film upon contact but quickly slide apart without merging, typically because of insufficient contact duration for drainage;
- **Coalescence:** After contact, the liquid film between the bubbles drains and ruptures, allowing the bubbles to combine into a single bubble.

To clearly illustrate the relationship between observed behaviors and the proposed physical criteria, Fig. 9 presents a flowchart of the interaction evolution. The horizontal axis represents the bubble interaction stages (approach and contact), while the vertical axis shows the three behavioral pathways.

Table 4a summarizes the physical criteria corresponding to each type of bubble interaction behavior. Additionally, Table 4b shows the data extracted from the experiment, which outlines the data quantities obtained under different experimental configurations. This data serves as a foundation for assessing model accuracy and interpreting the mechanisms of bubble coalescence.

4. Experimental results

We conduct statistical validation and preliminary verification of the proposed coalescence model through single and dual-nozzle release experiments, quantifying model prediction accuracy under controlled gas emission scenarios. We also analyze the parameter sensitivity, which identifies dominant factors governing bubble coalescence in buoyancy-driven plumes.

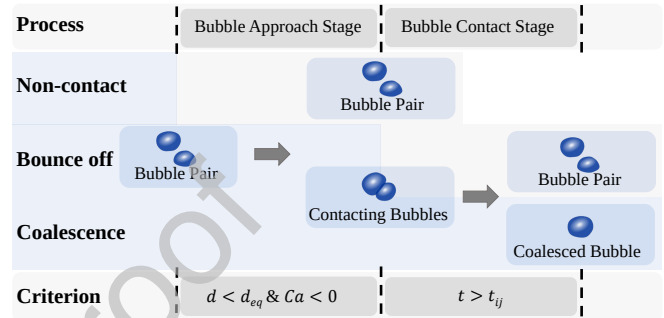


Fig. 9: Schematic of bubble interaction stages and corresponding behavioral transitions.

Table 4

Bubble Interaction Behaviors and Criteria

(a) Criteria for bubble interaction behaviors

Interaction Behaviors	Coalescence Criteria
Non-contact	$d > d_{eq}$ or $Ca > 0$
Bounce off	$d < d_{eq}$, $Ca < 0$, and $t < t_{ij}$
Coalescence	$d < d_{eq}$, $Ca < 0$, and $t > t_{ij}$

(b) Count of interaction events under different nozzle configurations

Type	Coalescence	Non-Contact	Bounce off
Single-nozzle	206	300	300
Dual-nozzle	335	300	300

4.1. Condition verification

To validate the criteria for bubble coalescence proposed in Section 2, we utilized bubble trajectory data obtained from both single- and dual-nozzle experiments to extract key dynamic parameters, including radius, velocity, contact time, and relative distance. These parameters were subsequently substituted into the theoretical framework to determine whether the coalescence conditions were satisfied. Fig. 10 shows a schematic of the verification process, which illustrates how experimental measurements are integrated into each criterion model and how the coalescence outcomes are determined based on the theoretical predictions.

Buoyancy-driven bubble coalescence model

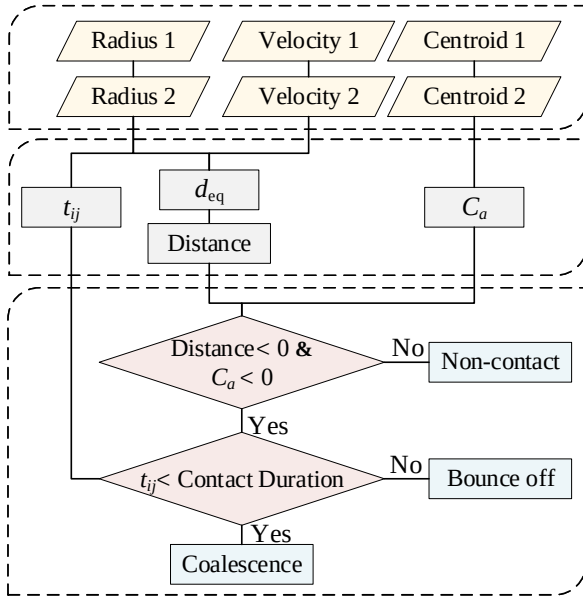


Fig. 10: Model verification process.

In the model criterion verification, 506 single-nozzle and 635 dual-nozzle contact events were recorded in the experiments, of which 206 and 335 were post-contact coalescence events, respectively. For each contact event, we select the frame immediately before bubble contact. From these frames, bubble radius and velocity are extracted and used in the model to compute the critical coalescence distance, attraction coefficient, and minimum drainage time.

For non-contact bubble cases, 300 single-nozzle and 300 dual-nozzle events are analyzed. A frame-by-frame evaluation is performed over the entire rising trajectory of each bubble pair. At each time step, instantaneous parameters such as radius, velocity, inter-bubble spacing, and collision angle are computed to evaluate dynamically whether the conditions for coalescence were met.

Figure 11 shows the experimental statistical results, which demonstrate the consistency between model predictions and observed outcomes under various criteria. Figure 11 (a) illustrates the deviation between the measured inter-bubble distance and the theoretically predicted critical coalescence distance. All coalesced bubble pairs fall within a negative deviation range (-2 to -9 mm), while non-contact bubbles exhibit a broader distribution (-7 to 8 mm), with a predominant shift towards positive values. This suggests that the distance condition is not the sole determinant for bubble contact, as other factors influence whether the bubbles actually make contact.

Fig. 11 (b) shows that the attraction coefficients for coalesced bubbles are predominantly negative, displaying a distinctly skewed distribution, which suggests the presence of attractive hydrodynamic interactions. In contrast, the attraction coefficients for non-contact bubbles are narrowly distributed around positive values, indicating weak repulsion and limited interaction.

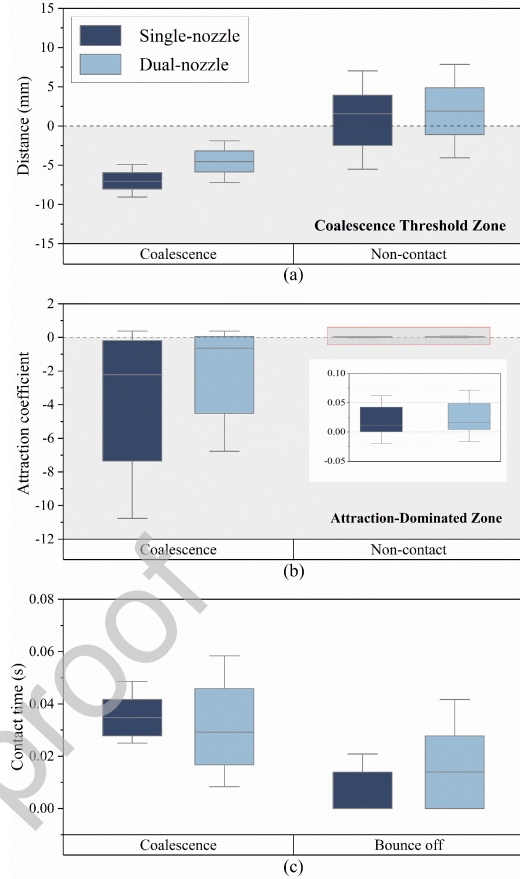


Fig. 11: Critical parameter comparison in bubble coalescence theory: (a) The distance difference between the coalesced and non-contact bubbles; (b) The attraction coefficient between the coalesced and non-contact bubbles; (c) The contact time between coalesced bubbles and bounce off bubble pairs.

Fig. 11 (c) reveals that the contact duration in coalescence events is significantly longer than that in non-contact bubble events. Additionally, the contact time distribution is broader under the dual-nozzle condition, reflecting enhanced turbulence-induced perturbations that influence the film drainage process. Collectively, the trends shown in Fig. 11 are in agreement with theoretical expectations, affirming the robustness and applicability of the proposed coalescence model across a range of dynamic parameter conditions.

4.2. Mechanism analysis

To further investigate the mechanisms governing bubble coalescence, we systematically analyzed the critical conditions of bubbles under varying bubble sizes and velocities (Fig. 12). The dashed line represents the theoretical boundary predicted by the linearized model, while the scattered data points represent coalescence and non-contact states observed under different experimental conditions. Each dataset corresponds to five nozzle specifications that controlled bubble size during gas release.

Buoyancy-driven bubble coalescence model

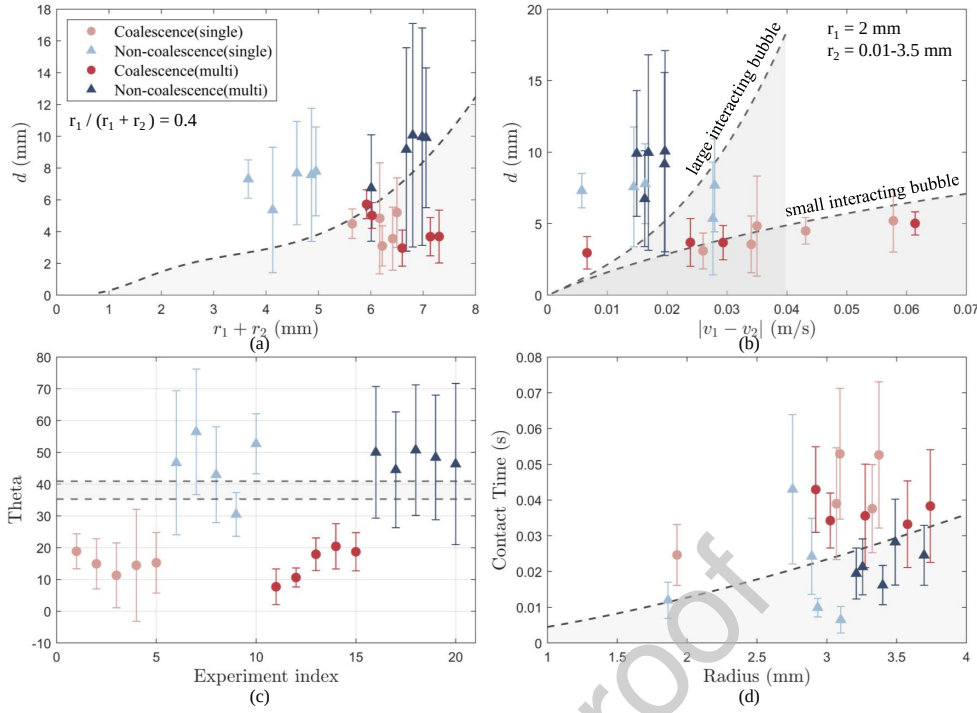


Fig. 12: Experimental-theoretical correlation of bubble coalescence. Dashed curves denote theoretical predictions, while symbols represent experimental observations for single- and dual-nozzle configurations under coalescence and non-coalescence conditions (where, $k_1 = 1^{-4}$, $k_2 = 0.5$, $k_3 = 142$). (a) Equivalent critical distance d_{eq} as a function of the total bubble radius $r_1 + r_2$; (b) Dependence of the equivalent critical distance d_{eq} on the relative velocity difference $|v_1 - v_2|$; (c) Distribution of collision angles θ compared with the theoretically predicted angular range; (d) Contact time as a function of bubble radius, compared with the theoretical drainage model.

Fig. 12 (a–b) illustrates the distribution of the merging criteria under varying bubble radii and relative velocity conditions. The theoretical curves define the threshold boundaries that indicate whether the critical separation distance required for merging is satisfied for a given combination of bubble sizes and relative velocities. It should be emphasized that this figure is not intended to demonstrate a strict monotonic dependence of the critical distance on the governing parameters, but rather to assess the applicability of the theoretical criteria under experimental conditions.

Despite the inherent dispersion in the experimental data, most coalescence events are observed to cluster near the theoretical threshold or within the predicted merging region, while non-contact events predominantly lie outside the threshold boundary. This distribution confirms that the proposed criterion is effective in discriminating whether the merging condition is met.

For Fig. 12 (a), the dashed line is calculated by keeping the bubble radius ratio $r_1/r_2 = 0.4$ constant, while varying the total radius $r_{sum} = r_1 + r_2$. The relative velocity Δv is computed based on the terminal velocity difference of bubbles with different radii.

Additionally, two theoretical prediction curves are presented in Fig. 12 (b) for the same relative velocity. This feature arises from the construction of the theoretical boundary, where the size of one bubble is kept constant ($r_1 = 2$ mm) while the size of the interacting bubble is varied. As a

result, identical relative velocity differences can correspond to different combinations of bubble sizes, leading to distinct critical separation distances. Therefore, the dual curves do not reflect uncertainty in the model predictions, but rather the coupled influence of relative velocity and bubble size on the merging threshold.

Figure 12(c) shows that, for nearly all coalescing bubble pairs, the collision angles are substantially smaller than the critical angle range predicted by the theory. It should be emphasized that the critical angle defined in the theoretical model is not intended to predict the collision angle at which coalescence actually occurs, but rather to establish an upper bound within which the attractive mechanism can operate. In the present study, the theoretical critical angle spans $35.3^\circ - 40.9^\circ$, corresponding to a geometric separation ratio of $S \approx 2.0 - 10.0$. This range, therefore, represents the allowable angular window for the existence of attractive interactions, rather than the realized collision angle during coalescence.

Experimentally, the collision angles of coalescing bubble pairs are predominantly confined to much smaller values, typically below 20° . This observation suggests that, during bubble ascent and mutual approach, the attractive interaction is accompanied by trajectory bending and angular compression. Such behavior is likely associated with wake-induced asymmetry in the surrounding flow field and the resulting additional attraction, which progressively guides the bubbles

Buoyancy-driven bubble coalescence model

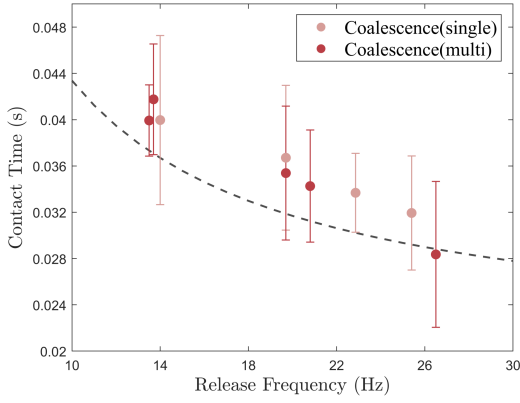


Fig. 13: Effect of bubble release frequency on coalescence contact time.

toward a more coaxial contact configuration before contact. The role of wake effects and their influence on collision geometry are discussed further in the following section.

Fig. 12 (d) shows that contact time increases with bubble radius, consistent with the liquid film drainage model. Notably, one group of non-contact bubbles in the single-nozzle configuration shows unusually long contact times. This anomaly may result from reduced turbulence dissipation at low gas flow rates, which delays the drainage and rupture of the intervening liquid film, thereby preventing coalescence.

Fig. 13 presents the experimental results showing the variation in bubble release frequency achieved by controlling different flow rates (20, 40, 60, 80, 100 mL/min) while keeping the nozzle diameter at 0.86 mm. The results reveal a clear negative correlation: as the release frequency decreases, the contact time between bubbles increases significantly. This trend is consistent with the observations from the single-nozzle configuration (Fig. 12(d)), where lower turbulence levels, resulting from reduced flow rates, extend the liquid film drainage period and delay coalescence. In contrast, higher release frequencies enhance turbulent interactions among bubbles, promoting faster drainage of the liquid film and thus shortening the coalescence time.

4.3. Model accuracy evaluation

To evaluate the reliability of the proposed model, this section presents the statistical accuracy of predictions based on the criterion indicators computed as described above. For merging events, a prediction is considered correct if the model satisfies any of the following conditions: (i) $d_{eq} > d$; (ii) $Ca < 0$; (iii) both of the above conditions are met simultaneously. The prediction accuracy for merging events is defined as:

$$A_{coal} = \frac{N_{match, coal}}{N_{obs, coal}}, \quad (25)$$

where $N_{match, coal}$ denotes the number of correctly predicted coalescence events, and $N_{obs, coal}$ is the total number of observed coalescence events.

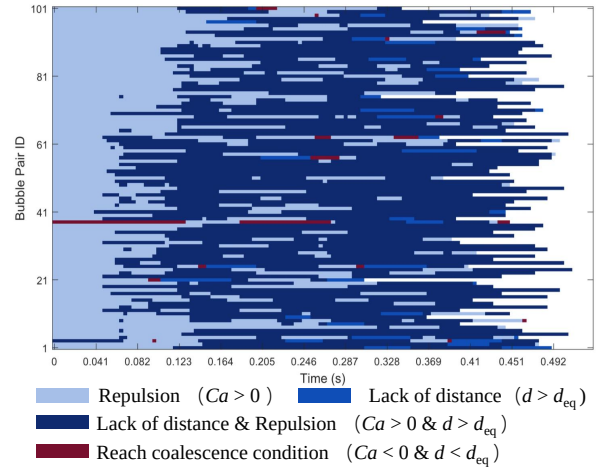


Fig. 14: Frame-by-frame verification matrix for 100 non-contact bubble pairs. Each row represents a set of bubble pairs, with the horizontal axis showing time steps during ascent. Blue indicates model predictions matching experimental observations (true negative): light blue for at least one non-contact criterion met, dark blue for both criteria met. Red areas represent false positives, where the model's criteria are met but no coalescence is observed experimentally.

For non-contact events, each frame of the bubble rising is analyzed individually. If the model's prediction for a given frame matches the experimental observation (i.e., no coalescence occurs), that frame is considered correctly predicted. The prediction accuracy for non-contact events is then calculated as:

$$A_{nc, frame} = \frac{N_{match, frame}}{N_{total, frame}}, \quad (26)$$

where $N_{match, frame}$ is the number of correctly predicted frames, and $N_{total, frame}$ is the total number of frames in the non-contact event.

Fig. 14 illustrates the prediction results for 100 bubble pairs in non-coalescence events. The horizontal axis represents the time steps during the bubble rising process, while the vertical axis denotes the bubble pair index. Different colors indicate the outcomes of the criteria: light blue corresponds to cases where at least one criterion is not satisfied, dark blue indicates that neither criterion is satisfied, and red represents cases where the coalescence criterion is met, but no coalescence is observed experimentally, corresponding to false-positive predictions.

Fig. 14 quantitatively evaluates the model's prediction accuracy for 100 non-contact bubble pairs through a time series matrix. The vertical axis represents individual bubble pair samples, while the horizontal axis corresponds to the time steps along the bubble's entire ascent. The blue region indicates true negative results, where the model's predictions align with experimental observations. Specifically, the light blue area shows that at least one physical criterion (i.e., $d_{eq} \leq d$ or $Ca \geq 0$) satisfies the non-coalescence condition for the bubble pair at that time step, while the dark blue area indicates both criteria are not met for coalescence.

Regardless of shading, the blue regions reflect agreement between the theoretical model and experimental outcomes. In contrast, the red areas indicate false positive predictions, where the model suggests coalescence but no merging is observed in the experiment.

To evaluate the reliability of the proposed criterion model under different interaction scenarios, Table 5 summarizes the consistency rates of the distance criterion, attraction criterion, and combined criterion for predicting coalescence and non-contact events under single-nozzle and dual-nozzle experimental conditions. Comparing the performance of these criteria in the two event types provides insight into the distinct roles of the underlying physical constraints in bubble interaction determination.

For coalescence events, the distance criterion exhibits consistently high agreement (exceeding 95%) under both experimental configurations. This outcome is physically intuitive, as geometric proximity within a critical distance is a necessary prerequisite for bubble coalescence. However, when the attraction criterion is incorporated to form the combined criterion, the overall consistency decreases slightly (approximately 81–83%), indicating that the attraction condition—related to the relative bubble orientation and wake-induced interaction—acts as a more restrictive constraint on coalescence prediction.

Notably, although the experimentally observed collision angles of coalescing bubbles are predominantly concentrated within a narrow, near-coaxial range and are generally much smaller than the theoretical critical attraction angle, the attraction criterion still limits the model's predictive coverage. This behavior can be attributed to transient bubble deformation or trajectory oscillations during the approach process, which may cause the instantaneous configuration to deviate from the idealized steady attraction conditions assumed in the simplified model. This discrepancy highlights the influence of interaction stability on coalescence outcomes.

For non-contact events, the distance criterion alone yields a substantially lower consistency rate (64.0% under single-nozzle conditions), demonstrating that reliance on geometric distance alone leads to frequent false-positive predictions. During bubble ascent, transient close approaches frequently occur; however, many of these bubble pairs remain in a hydrodynamically repulsive state and do not ultimately make contact or coalesce.

In contrast, the combined criterion achieves a markedly higher consistency rate (exceeding 90%) for non-contact events. This improvement indicates that the attraction criterion enables the model to effectively distinguish bubble pairs that are geometrically close but dynamically repulsive, thereby filtering out misclassifications caused solely by transient proximity. These results underscore that geometric distance alone is insufficient to characterize bubble interactions and that dynamic interaction mechanisms play a decisive role.

Overall, the distance criterion provides a necessary condition with high coverage for coalescence events, whereas

Table 5

Consistency rates of different prediction criteria for bubble interactions

Case	Distance & Attraction & Combined	Time	All
Coalescence (single)	95.5 & 82.5 & 81.1%	89.7%	72.7%
Coalescence (Dual)	97.4 & 84.5 & 83.4%	87.1%	72.6%
Non-contact (single)	64.0 & 90.3 & 96.5%	-	96.5%
Non-contact (Dual)	72.0 & 81.0 & 90.3%	-	90.3%

the attraction criterion is essential for discriminating between attractive and repulsive interaction regimes. It should be emphasized that the dynamic model employed in this study is based on idealized assumptions. In practical flows, wake effects, bubble deformation, and local flow disturbances may influence interaction behavior, as reflected in the above analysis. These factors and the associated limitations of the present model are further discussed in the following section.

5. Discussion and conclusions

5.1. Discussion

To clarify the limitations associated with the simplified theoretical framework, the experimental conditions are first examined within a flow-regime map defined by key dimensionless parameters (Fig. 15). The shape and dynamical behavior of rising bubbles are primarily governed by the Eötvös number (Eu), Reynolds number (Re), and Galilei number (Ga), which respectively characterize the relative importance of buoyancy to surface tension forces (governing deformation), inertial to viscous forces (governing wake structure), and gravity-driven inertia to viscous dissipation (governing characteristic flow scales) [28, 51]. The definitions and calculation procedures of these dimensionless numbers are provided in the Supplementary Material.

As shown in Fig. 15, the experimental conditions of this study are mainly distributed within the parameter ranges of $0.5 \lesssim Eu \lesssim 3.8$ and $50 \lesssim Re \lesssim 1000$, corresponding to the ellipsoidal and wobbling regimes in the classical bubble dynamics phase diagram. Within this regime, bubbles typically exhibit pronounced non-spherical deformation and unsteady shape oscillations, and are often accompanied by wake-induced flow disturbances. These features highlight intrinsic limitations of models based on spherical geometry and steady interaction assumptions when applied to the present experimental conditions.

Fig. 16 presents the scatter distributions of the bubble surface distance d , collision angle θ , and contact duration t as functions of Re and Eu for both coalescence and non-coalescence events. Rather than indicating simple monotonic dependencies on individual dimensionless parameters,

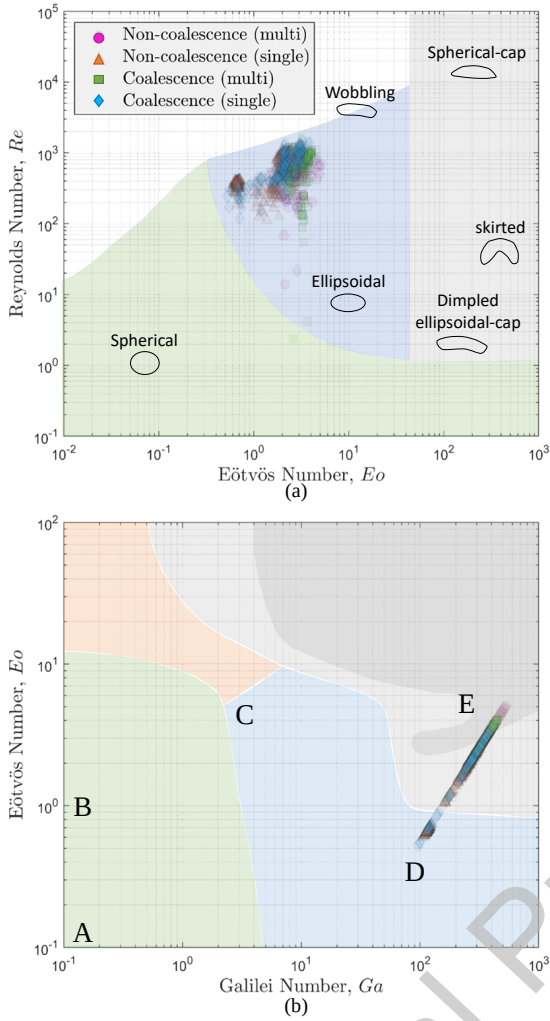


Fig. 15: Bubble Flow Regime Map and Experimental Data Distribution on $Eo - Re$ and $Ga - Eo$ Planes: (a) $Eo - Re$ Plane: The background flow regime boundaries are adapted from the classic work by Grace et al. [28, 52], which are based on empirical observations and thus are somewhat arbitrary. (b) $Ga - Eo$ Plane: The background flow regime boundaries are modified from the phase diagram proposed by Tripathi et al. [53], derived from their numerical study of unsteady bubble dynamics.

these distributions provide a statistical basis for discussing how bubble deformation, wake effects, and local flow disturbances jointly modulate bubble interactions.

5.1.1. Bubble deformation

From the scatter distributions in Fig. 16, it is evident that for bubble pairs undergoing coalescence, neither bubble surface distance d nor the contact duration t shows a meaningful linear correlation with Re or Eo , as indicated by consistently low coefficients of determination ($R^2 < 0.1$). This observation suggests that when bubble deformation becomes significant, the interaction range cannot be adequately described using global dimensionless parameters alone, consistent with previous studies on deformed bubble interactions [51].

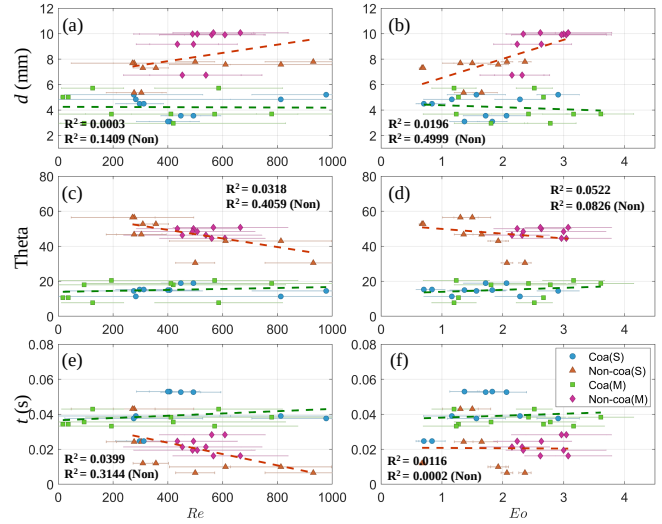


Fig. 16: Scatter distributions of the bubble surface distance d , contact duration t , and collision angle θ as functions of the Reynolds number Re and the Eötvös number Eo for coalescence and non-coalescence events. (a) d vs Re ; (b) d vs Eo ; (c) θ vs Re ; (d) θ vs Eo ; (e) t vs Re ; (f) t vs Eo .

In contrast, non-coalescence events exhibit a moderate but statistically scattered positive trend between Eo and d (Fig. 16(b)). As Eo increases, reduced surface tension constraints promote bubble flattening and deviations from ideal spherical geometry [51]. Such deformation may modify near-field flow structures and enhance local hydrodynamic repulsion, effectively extending the separation distance at which interactions fail and resulting in a larger equivalent critical distance.

The contact duration t for non-coalescing bubble pairs shows substantial scatter without a clear monotonic dependence on Eo (Fig. 16(f)). This dispersion is likely related to deformation-induced variability in contact geometry and local interfacial configuration, which introduces pronounced transient and non-uniform characteristics during the contact process. Under such conditions, t is no longer governed by a single geometric or dynamic scale.

Overall, these results indicate that bubble deformation imposes additional dynamical constraints on the coalescence process by altering near-field geometry and local flow structures. Such effects are not readily captured by simplified criteria based solely on global parameters, highlighting the need for morphology-aware corrections in future coalescence models.

5.1.2. Wake effects

In addition to deformation effects, wake-induced interactions and vortex structures may also contribute to the observed discrepancies between theory and experiment. The equivalent-radius approximation used in the model neglects possible wake asymmetry arising from bubble size differences, while at sufficiently high Re the wake becomes open and unsteady, with vortex shedding and wobbling

motions [28, 46], which may further modulate bubble interactions.

(i) Attraction angle: The experimental results indicate that bubble pairs undergoing coalescence are confined to a relatively narrow range of collision angles, whereas non-coalescence events show a weak tendency toward smaller θ at higher Re (Fig. 16(c)). This behavior may reflect the coexistence of two competing wake-related effects. On the one hand, enhanced wake entrainment at higher Re could promote partial alignment of the trailing bubble toward the ascent axis of the leading bubble, leading to a statistical compression of the approach geometry. On the other hand, in the high- Re regime, vortex shedding in open wakes introduces unsteady lateral forces that can perturb the bubble trajectories. When the approach path deviates modestly from coaxial alignment, these fluctuations may be sufficient to deflect the bubbles away from the capture region, thereby suppressing coalescence [54]. From this perspective, the narrow angular window associated with coalescence events is more appropriately viewed as a consequence of hydrodynamic filtering of successful coalescence events, rather than as a direct or monotonic dependence of θ on Re .

(ii) Contact time: A similar interpretation can be proposed for the contact time. For non-coalescence events, the measured contact duration exhibits a weak decreasing tendency with increasing Re (Fig. 16(e)). In an open and increasingly unsteady wake, enhanced shear fluctuations and lateral perturbations may promote intermittent separation during the contact stage, effectively shortening the continuous contact duration identified from image sequences. By contrast, bubble pairs that successfully coalesce tend to display contact times clustered within a narrower range, suggesting that coalescence requires not only a near-coaxial approach geometry but also a sufficiently sustained contact period to allow for liquid film drainage and destabilization [28, 35]. Coalescence may therefore be interpreted as a threshold screening process jointly constrained by geometric and temporal windows, rather than as a process governed by Re alone.

It should be noted that the trends observed for non-coalescence events with increasing Re in Fig. 16 are weak and accompanied by substantial scatter. Accordingly, the above interpretations are intended to highlight physically plausible tendencies associated with wake unsteadiness, rather than to establish definitive quantitative relationships.

5.1.3. Weber number

The Weber number (We) is a key dimensionless quantity that characterizes the balance between inertial forces and surface tension during bubble collisions. Previous studies have frequently used the Weber number to investigate bubble aggregation behaviors and determine the critical We value [26, 27]. In this experiment, we also calculated and statistically analyzed the We values of bubble pairs and conducted a comparative analysis.

The statistical results (Figure 17) reveal significant differences in the distribution of We between coalescence and

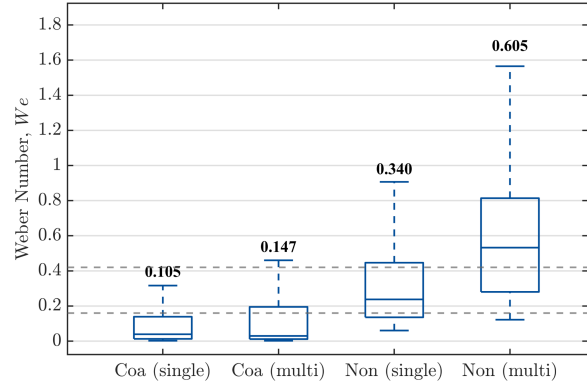


Fig. 17: Weber Number Distribution and Critical Threshold Comparison.

bounce-off events. For coalescence events, the average We values under single-nozzle and dual-nozzle conditions are 0.105 and 0.147, respectively. Referring to the study by Yang et al. [26], the critical We for head-on collisions in quasi-static water is approximately 0.16. In this study, the majority of coalescence events exhibited We values below 0.5, with the mean values closely aligning with the classical theoretical threshold. Notably, the We distribution for the dual-nozzle coalescence group is higher than that of the single-nozzle group. This can be attributed to the influence of eccentric collisions: in the dual-nozzle configuration, bubbles more frequently collide off-center. According to Yang et al.'s theory, eccentric collisions can elevate the critical We to 0.42.

In contrast, for bounce-off events, the average We values under single-nozzle and dual-nozzle configurations are 0.340 and 0.605, respectively, which are significantly higher than those observed in coalescence events. Furthermore, statistical analysis indicates that the We values for bubbles in the dual-nozzle configuration are not only larger but also exhibit a broader distribution range. This may be attributed to the increased likelihood of non-perpendicular collisions in the dual-nozzle configuration.

The above analysis reveals that neither the classic Re , Eu nor We , which are directly related to the collision dynamics, can serve as a single, universal criterion to reliably distinguish between aggregation and non-aggregation events. This highlights the limitations of using a single global dimensionless number as a criterion for bubble aggregation.

To address this limitation, this study posits that the aggregation process fundamentally acts as a screening mechanism, constrained by the coupled influence of multiple physical fields. The unified criterion for bubble coalescence can be expressed as:

$$C \implies (d < d_{eq}) \cap (C_a < 0) \cap (t > t_{ij}) \quad (27)$$

This criterion demonstrates that the conditions for bubble aggregation are not determined by a single global dimensionless number (such as Re or Eu), but instead arise from the combined effect of the three sub-physical criteria discussed above.

5.2. Conclusions

In this paper, we propose a threshold model for predicting bubble coalescence in buoyancy-driven flow, incorporating three critical criteria: (i) distance criterion, (ii) attraction coefficient, and (iii) drainage time criterion. These criteria are derived based on physical principles, including energy balance, hydrodynamic interaction, and liquid film drainage theory. The model was linearized to facilitate sensitivity analysis and experimental comparison under varying release conditions.

The proposed model was validated through single- and dual-nozzle experiments, where bubble trajectories were analyzed to assess coalescence outcomes. Experimental validation under both single- and dual-nozzle configurations confirmed the predictive capability of the proposed model, with average consistency rates of 84.6 % for coalescence events and 81.5 % for non-contact events under the combined criterion. The model effectively captures the dominant trends of coalescence dynamics and reveals key influences of bubble size, relative velocity, and interaction geometry within the present experimental parameter range.

While the proposed model provides valuable insights into the dynamics of bubble coalescence, it should be noted that it simplifies several key physical processes, such as bubble deformation and wake-induced forces. Despite extensive discussion of these effects, we have yet to fully capture the detailed mechanisms governing coalescence under all conditions. This is due to the non-linear and complex nature of these interactions, which cannot be easily modeled using global dimensionless parameters alone.

Future work will seek to incorporate bubble deformation, wake interactions, and stochastic turbulence to further improve the model's accuracy over a broader range of flow conditions. The proposed framework is intended primarily as a physically interpretable bubble coalescence model for numerical simulations. Under practical conditions where detailed bubble-scale measurements are unavailable, the variables required by Eq. 27 can be obtained from the numerical framework. In bubble-resolved or Eulerian–Lagrangian simulations, the bubble surface distance d and interaction geometry can be evaluated from the simulated instantaneous bubble positions, sizes, and velocities, from which the attraction coefficient C_a can be calculated. The contact duration t can be determined from the pairwise interaction history [55]. In population balance models, where individual bubble-pair interactions are not explicitly resolved, these quantities can be represented through statistical closure relations and incorporated into the coalescence efficiency or coalescence kernel [9]. Therefore, the present framework can provide a physics-based coalescence closure for coupling bubble coalescence with transport and dissolution in large-scale underwater bubble-plume simulations.

Symbol	Description
r, r_1, r_2	Equivalent radius of a single bubble, subscripts 1, 2 distinguish interacting bubbles (m)
r_{ij}	Effective radius of a bubble pair (m)
$\vec{v}, \vec{v}_1, \vec{v}_2$	Velocity of a single bubble, subscripts 1, 2 distinguish interacting bubbles (m/s)
d	Surface distance between bubbles (m)
d_{eq}	Maximum critical distance threshold under energy balance (surface distance) (m)
$W_{driving}$	Driving work (N)
$W_{resisting}$	Resisting work (N)
$W_{viscous}$	viscous dissipation work (N)
$W_{tension}$	Surface tension resistance work (N)
KE_{net}	kinetic energy (N)
m_{eff}	Effective (reduced) inertial mass of the interacting system (kg)
ρ, μ, σ	Density, dynamic viscosity, and surface tension (kg/m^3), ($\text{Pa}\cdot\text{s}$), (N/m)
θ	Angle between the vector connecting bubble centers and the vertical upward direction ($^\circ$)
θ_{crit}	Critical attraction angle ($^\circ$)
F_B, F_D, F_L	Buoyancy force, drag force, and lift force (N)
F_{VM}, F_G	Virtual mass force and gravitational force (N)
C_a	Attraction coefficient
M^*, N^*	Interaction parameters expressed as functions of normalized separation
S	Dimensionless distance, $S = \frac{d+2r_{ij}}{r_{ij}}$
t	Actual bubble contact time observed in the experiment (s)
t_{ij}	Drainage time, theoretical minimum time for liquid film rupture (s)
h_0, h_f	Initial thickness and critical rupture thickness (m)
w, h, l	Model outputs a bounding box defined by its top-left corner (x_k, y_k) and pixel dimensions (w_k, h_k) (pixels)
α_k	Corresponding calibration factor
$\mathbf{C}$	Bubble centroid, $\mathbf{C} = (C_x, C_y, C_z)$ (m)
Δt	Time interval between consecutive frames (s)

1. Nomenclature

2. Algorithm Details

CRedit authorship contribution statement

Xin Ren: Methodology, Formal analysis, Visualization, Writing – original draft. **Jianghui Li:** Conceptualization; Methodology; Formal analysis; Software, Visualization, Resources; Writing - Review & Editing; Supervision; Project administration; Funding acquisition. **Tianyu Xiao:** Resources; Visualization. **Daolong Zhou:** Resources; Visualization. **Xuheng Yang:** Resources. **Fengling Yu:** Supervision. **Xiaokang Zhang:** Resources.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Algorithm 1 Bubble Parameter Calculation via Dual-View Fusion

Require: Dual-view video frames F_1, F_2 , Acoustic data, Calibration parameters $\{O_1, O_2, s_1, s_2\}$, Time sequence $t \in \mathbb{R}^T$

Ensure: Bubble parameters $\{\text{Centroid}, \mathbf{R}, \mathbf{V}\}$,

Stage 0: Detection & Tracking

for view $k \leftarrow 1$ to 2 **do**

Detect bubbles using YOLO v8:

$B_k \leftarrow \text{YOLO V8}(F_k) \triangleright$ Outputs $(x, y, w, h, \text{conf})$

Track bubbles using SORT:

$A_k \leftarrow \text{SORT}(B_k) \triangleright$ Kalman filter + Hungarian

matching

end for

Stage 1: Multi-View Fusion

for time frame $\tau \leftarrow 1$ to T **do**

3D coordinate fusion:

$\mathbf{p}_\tau \leftarrow (\mathbf{c}_1^m, \mathbf{c}_2^m)$

Centroid calculation:

$\text{Centroid}_{\tau.x} \leftarrow \mathbf{c}_1^m.x$

$\text{Centroid}_{\tau.y} \leftarrow \mathbf{c}_2^m.x$

$\text{Centroid}_{\tau.z} \leftarrow \frac{\mathbf{c}_1^m.y + \mathbf{c}_2^m.y}{2}$

$\triangleright$ Outputs $\text{Centroid}(x, y, z)$

Radius calculation:

$\mathbf{R}_\tau.\text{length} \leftarrow \mathbf{c}_1^m.w$

$\mathbf{R}_\tau.\text{width} \leftarrow \mathbf{c}_2^m.w$

$\mathbf{R}_\tau.\text{height} \leftarrow \frac{\mathbf{c}_1^m.h + \mathbf{c}_2^m.h}{2}$

$\triangleright$ Outputs $\mathbf{R}(l, w, h)$

Velocity calculation:

$\mathbf{V}_\tau \leftarrow \frac{\mathbf{p}_{\tau+1} - \mathbf{p}_{\tau-1}}{t_{\tau+1} - t_{\tau-1}}$

$\triangleright$ Outputs $\mathbf{V}$

end for

Acknowledgements

Thanks to the National Major Science and Technology Projects of China (No.2024ZD1406600, 2024ZD1406604, 2024ZD1406604-11), the Natural Science Foundation of Xiamen Municipality (No.3502Z202373006), and the MEL Internal Program (No.MELRI2303) for supporting this research financially.

References

- [1] Y. Feng, S. Wang, and L. An, "Investigation of bubble coalescence and evolution performance on electrode surface," *Chemical Engineering Science*, vol. 309, p. 121443, 2025.
- [2] Z. Zhang, R. Wu, and C. Zhao, "Pore-scale study of buoyancy-driven gas bubble migration, breakup, trapping, and coalescence in the near injection region of liquid saturated porous media," *Industrial & Engineering Chemistry Research*, vol. 60, no. 33, pp. 12419–12428, 2021.
- [3] N. Kantarci, F. Borak, and K. O. Ulgen, "Bubble column reactors," *Process biochemistry*, vol. 40, no. 7, pp. 2263–2283, 2005.
- [4] G. Allan and J. Woodcock, "A review of the flotation of native gold and electrum," *Minerals Engineering*, vol. 14, no. 9, pp. 931–962, 2001.
- [5] Ş. B. Aydın and A. Gül, "Kinetic modelling and optimization of flotation process of electrum," *Physicochemical Problems of Mineral Processing*, vol. 57, no. 6, pp. 80–94, 2021.
- [6] A. Poulikkas, "Effects of two-phase liquid-gas flow on the performance of nuclear reactor cooling pumps," *Progress in Nuclear Energy*, vol. 42, no. 1, pp. 3–10, 2003.
- [7] H.-M. Prasser, M. Beyer, H. Carl, S. Gregor, D. Lucas, H. Pietruske, P. Schütz, and F.-P. Weiss, "Evolution of the structure of a gas-liquid two-phase flow in a large vertical pipe," *Nuclear Engineering and Design*, vol. 237, no. 15-17, pp. 1848–1861, 2007.
- [8] J. Li, B. Roche, J. M. Bull, P. R. White, T. G. Leighton, G. Provenzano, M. Dewar, and T. J. Henstock, "Broadband acoustic inversion for gas flux quantification—application to a methane plume at scanner pockmark, central North Sea," *Journal of Geophysical Research: Oceans*, vol. 125, no. 9, p. e2020JC016360, 2020.
- [9] F. Lehr, M. Millies, and D. Mewes, "Bubble-size distributions and flow fields in bubble columns," *AIChE Journal*, vol. 48, no. 11, pp. 2426–2443, 2002.
- [10] S. Das, "Development of a coalescence model due to turbulence for the population balance equation," *Chemical Engineering Science*, vol. 137, pp. 22–30, 2015.
- [11] I. P. on Climate Change (IPCC), *Climate Change 2021 – The Physical Science Basis: Working Group I Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press, 2023.
- [12] C. D. Ruppel and J. D. Kessler, "The interaction of climate change and methane hydrates," *Reviews of Geophysics*, vol. 55, no. 1, pp. 126–168, 2017.
- [13] G. Rehder, I. Leifer, P. G. Brewer, G. Friederich, and E. T. Peltzer, "Controls on methane bubble dissolution inside and outside the hydrate stability field from open ocean field experiments and numerical modeling," *Marine Chemistry*, vol. 114, no. 1, pp. 19–30, 2009.
- [14] L. A. Levin, A. R. Baco, D. A. Bowden, A. Colaco, E. E. Cordes, M. R. Cunha, A. W. Demopoulos, J. Gobin, B. M. Grupe, J. Le, *et al.*, "Hydrothermal vents and methane seeps: rethinking the sphere of influence," *Frontiers in Marine Science*, vol. 3, p. 72, 2016.
- [15] A. Boetius, K. Ravensschlag, C. J. Schubert, D. Rickert, F. Widdel, A. Gieseke, R. Amann, B. B. Jørgensen, U. Witte, and O. Pfannkuche, "A marine microbial consortium apparently mediating anaerobic oxidation of methane," *Nature*, vol. 407, no. 6804, pp. 623–626, 2000.
- [16] J. Li, P. R. White, B. Roche, J. M. Bull, T. G. Leighton, J. W. Davis, and J. W. Fone, "Acoustic and optical determination of bubble size distributions—quantification of seabed gas emissions," *International Journal of Greenhouse Gas Control*, vol. 108, p. 103313, 2021.
- [17] Z. Hou, D. H. Bacon, D. W. Engel, G. Lin, Y. Fang, H. Ren, and Z. Fang, "Uncertainty analyses of CO2 plume expansion subsequent to wellbore CO2 leakage into aquifers," *International Journal of Greenhouse Gas Control*, vol. 27, pp. 69–80, 2014.
- [18] I. Leifer and R. K. Patro, "The bubble mechanism for methane transport from the shallow sea bed to the surface: A review and sensitivity study," *Continental Shelf Research*, vol. 22, no. 16, pp. 2409–2428, 2002. Gas in Marine Sediments: Contributions from the 5th International Conference organised by the Shallow Gas Group, Bologna, Italy, September 1998.
- [19] S. Das, "Development of a coalescence model due to turbulence for the population balance equation," *Chemical Engineering Science*, vol. 137, pp. 22–30, 2015.
- [20] S. C. Ozan, J. Solsvik, and H. A. Jakobsen, "A bubble coalescence kernel combining the characteristics of the film drainage, energy, and critical velocity models," *Chemical Engineering Science*, vol. 269, p. 118458, 2023.
- [21] M. E. Federle and R. Zenit, "Physical conditions for droplet-bubble coalescence in flotation," *International Journal of Multiphase Flow*, vol. 179, p. 104933, 2024.
- [22] E. Klaseboer, J. Chevaillier, C. Gourdon, and O. Masbernat, "Film drainage between colliding drops at constant approach velocity: Experiments and modeling," *Journal of Colloid and Interface Science*, vol. 229, no. 1, pp. 274–285, 2000.

- [23] Y. Liao and D. Lucas, "A literature review on mechanisms and models for the coalescence process of fluid particles," *Chemical Engineering Science*, vol. 65, no. 10, pp. 2851–2864, 2010.
- [24] T. Wang, J. Wang, and Y. Jin, "Theoretical prediction of flow regime transition in bubble columns by the population balance model," *Chemical Engineering Science*, vol. 60, no. 22, pp. 6199–6209, 2005. 7th International Conference on Gas-Liquid and Gas-Liquid-Solid Reactor Engineering.
- [25] M. J. Prince and H. W. Blanch, "Bubble coalescence and break-up in air-sparged bubble columns," *AIChE journal*, vol. 36, no. 10, pp. 1485–1499, 1990.
- [26] W. Yang, Z. Luo, Q. Lai, and Z. Zou, "Study on bubble coalescence and bouncing behaviors upon off-center collision in quiescent water," *Experimental Thermal and Fluid Science*, vol. 104, pp. 199–208, 2019.
- [27] P. C. Duineveld, "Bouncing and coalescence of bubble pairs rising at high reynolds number in pure water or aqueous surfactant solutions," *Springer Netherlands*, 1998.
- [28] R. Clift, J. R. Grace, and M. E. Weber, "Bubbles, drops, and particles," *International Journal of Heat and Mass Transfer*, 1978.
- [29] S. K. Friedlander *et al.*, *Smoke, dust, and haze*, vol. 198. Oxford university press New York, 2000.
- [30] S. Kalkach-Navarro, R. Lahey Jr, and D. Drew, "Analysis of the bubbly/slugg flow regime transition," *Nuclear engineering and design*, vol. 151, no. 1, pp. 15–39, 1994.
- [31] G. Kocamustafaogullari and M. Ishii, "Foundation of the interfacial area transport equation and its closure relations," *International Journal of Heat and Mass Transfer*, vol. 38, no. 3, pp. 481–493, 1995.
- [32] T. Liu and M. Schwarz, "CFD-based multiscale modelling of bubble-particle collision efficiency in a turbulent flotation cell," *Chemical Engineering Science*, vol. 64, no. 24, pp. 5287–5301, 2009.
- [33] P. Ghosh, *Colloid and interface science*. PHI Learning Pvt. Ltd., 2009.
- [34] R. Han, A. Zhang, S. Li, and Z. Zong, "Experimental and numerical study of the effects of a wall on the coalescence and collapse of bubble pairs," *Physics of Fluids*, vol. 30, no. 4, 2018.
- [35] J. N. Israelachvili, "4 - interactions involving polar molecules," in *Intermolecular and Surface Forces (Third Edition)* (J. N. Israelachvili, ed.), pp. 71–90, San Diego: Academic Press, third edition ed., 2011.
- [36] R. Pugh, "Foaming, foam films, antifoaming and defoaming," *Advances in colloid and interface science*, vol. 64, pp. 67–142, 1996.
- [37] C. Qu, Y. Yu, and J. Zhang, "Experimental study of bubbling regimes on submerged micro-orifices," *International Journal of Heat and Mass Transfer*, vol. 111, pp. 17–28, 2017.
- [38] L. Zhang and M. Shoji, "Aperiodic bubble formation from a submerged orifice," *Chemical Engineering Science*, vol. 56, no. 18, pp. 5371–5381, 2001.
- [39] X. Wang, M. Song, S. Li, and Y. Huang, "Dynamic behaviors of bubble formation on submerged micro-capillary under constant flow conditions," *Powder Technology*, vol. 417, p. 118258, 2023.
- [40] L. v. Wijngaarden, "The mean rise velocity of pairwise-interacting bubbles in liquid," *Journal of Fluid Mechanics*, vol. 251, p. 55–78, 1993.
- [41] D. W. Moore, "The boundary layer on a spherical gas bubble," *Journal of Fluid Mechanics*, vol. 16, no. 2, p. 161–176, 1963.
- [42] Y. HALLEZ and D. LEGENDRE, "Interaction between two spherical bubbles rising in a viscous liquid," *Journal of Fluid Mechanics*, vol. 673, p. 406–431, 2011.
- [43] J. B. Kok, "Dynamics of a pair of gas bubbles moving through liquid, part I," *European journal of mechanics. B-Fluids*, vol. 12, no. 4, pp. 515–540, 1993.
- [44] A. Chesters, "Modelling of coalescence processes in fluid-liquid dispersions: a review of current understanding," *Chemical engineering research and design*, vol. 69, no. A4, pp. 259–270, 1991.
- [45] C. A. Coulaloglou, *Dispersed Phase Interactions in an Agitated Flow Vessel*. Illinois Institute of Technology, 1975.
- [46] L. Liu, Y. Huang, Y. Liao, H. Zhang, and H. Yan, "Direct numerical simulation of side-by-side rising bubble coalescence behavior based on a weber number criterion," *International Journal of Multiphase Flow*, p. 105470, 2025.
- [47] J. Terven, D.-M. Córdova-Esparza, and J.-A. Romero-González, "A comprehensive review of YOLO architectures in computer vision: From YOLOv1 to YOLOv8 and YOLO-nas," *Machine learning and knowledge extraction*, vol. 5, no. 4, pp. 1680–1716, 2023.
- [48] R. E. Kalman, "A new approach to linear filtering and prediction problems," 1960.
- [49] A. Bewley, Z. Ge, L. Ott, F. Ramos, and B. Upcroft, "Simple online and realtime tracking," in *2016 IEEE international conference on image processing (ICIP)*, pp. 3464–3468, Ieee, 2016.
- [50] M. Li and L. Hu, "Experimental investigation of the behaviors of highly deformed bubbles produced by coaxial coalescence," *Experimental Thermal and Fluid Science*, vol. 117, p. 110114, 2020.
- [51] M. K. Tripathi, A. Premlata, K. C. Sahu, and R. Govindarajan, "Two initially spherical bubbles rising in quiescent liquid," *Physical Review Fluids*, vol. 2, no. 7, p. 073601, 2017.
- [52] J. R. Grace, T. Wairegi, and T. H. Nguyen, "Shapes and velocities of single drops and bubbles moving freely through immiscible liquids," *Chem.eng.res.des*, 1976.
- [53] M. K. Tripathi, K. C. Sahu, and R. Govindarajan, "Dynamics of an initially spherical bubble rising in quiescent liquid," *Nature communications*, vol. 6, no. 1, p. 6268, 2015.
- [54] A. Ghanavati, S. Khodadadi, M. H. Taleghani, M. Gorji-Bandpy, and D. D. Ganji, "Numerical simulation of the motion and interaction of bubble pair rising in a quiescent liquid," *Applied Ocean Research*, vol. 141, p. 103769, 2023.
- [55] B. Wang, I. Jun, S. A. Socolofsky, S. F. DiMarco, and J. D. Kessler, "Dynamics of gas bubbles from a submarine hydrocarbon seep within the hydrate stability zone," *Geophysical Research Letters*, vol. 47, no. 18, p. e2020GL089256, 2020.