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Simplified *p*-norm-like Constraint LMS Algorithm for Efficient Estimation of Underwater Acoustic Channels

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Abstract: Underwater acoustic channels are recognized for being one of the most difficult propagation media due to considerable difficulties such as: multipath, ambient noise, time-frequency selective fading. The exploitation of sparsity contained in underwater acoustic channels provides a potential solution to improve the performance of underwater acoustic channel estimation. Compared with the classic l_0 and l_1 norm constraint LMS algorithms, the *p*-norm-like (l_p) constraint LMS algorithm proposed in our previous investigation exhibits better sparsity exploitation performance at the presence of channel variations, as it enables the adaptability to the sparseness by tuning of p parameter. However, the decimal exponential calculation associated with the *p*-norm-like constraint LMS algorithm poses considerable limitations in practical application. In this paper, a simplified variant of the *p*-norm-like constraint LMS was proposed with the employment of Newton iteration method to approximate the decimal exponential calculation. Numerical simulations and the experimental results obtained in physical shallow water channels demonstrate the effectiveness of the proposed method compared to traditional norm constraint LMS algorithms.

Keywords: *p*-norm-like constraint; underwater acoustic channels; LMS algorithm; sparsity exploitation

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1 Introduction[1]

In recent years, there has been an increasing interest in underwater communications in many scientific, industrial, and research areas. However, due to the difficulties encountered in underwater acoustic (UWA) channels, digital communications through the usage of UWA channels are much more difficult than those in radio channel (Akyildiz *et al*., 2005; Chitre *et al*., 2008; Singer *et al*., 2009; Zhang *et al*., 2011). Due to the complicated and random nature of the propagation channel, the impulse response of which may extend over several tens or even a hundred milliseconds, severe inter symbol interference (ISI) will be caused at high transmission rates (Stojanovic, 2008). Thus, the R&D of high speed, large capacity, bandwidth-efficient digital communication systems are highly extremely challenging in underwater conditions. For the underwater acoustic research community, the channel estimation provides an effective solution to obtain the characteristics of underwater acoustic channels and also enable the adjustment of the algorithm parameter such as; the coefficients of channel equalizer accordingly.

As noted by Stojanovic (2005), the impulse response of the sparse systems typically consists of a large number of near-zero taps and with only a few large ones (Stojanovic, 2005), thus, providing significant potential of sparsity exploitation to improve the performance of system identification. The topic of sparsity exploitation has been extensively studied in many applications (Angelosante *et al*., 2010; Cotter and Rao, 2002; Gu *et al*., 2009; Jin *et al*., 2010; Kalouptsidis *et al*., 2011; Naylor *et al*., 2006; Shi K and Shi P, 2010, 2011). In Cotter and Rao (2002), a residual signal was used to maximize the correlation of a column of the mixture matrix, which is used to estimate channel taps one by one for sparsity utilization. However, this method has resulted in poor performance at the presence of significant inter-path interference or dense clusters. In a study conducted by Naylor *et al*., (2006), conventional LMS algorithm were modified to exploit the sparse characteristics of target systems, however, priori information of the unknown system is required.

The l_0-norm and l_1-norm constraint have been integrated into the cost function of the standard LMS algorithm to accelerate the convergence of near-zero coefficients in sparse system (Gu *et al*., 2009; Jin *et al*., 2010; Shi and Shi, 2010, 2011). It has been recognized that applying the concept of norm constraint for sparsity evaluation in LMS is helpful to improve performance. However, from the viewpoint of norm constraint, neither l_0-norm penalty LMS nor l_1-norm penalty LMS can provide adaptability to the characteristics of sparse systems (Shi and Shi, 2010, 2011), such as the time varying sparseness of underwater acoustic channels.

Time varying sparsity contained in underwater acoustic channels has already been addressed by using estimation methods based on two-dimensional delay-Doppler model (Li and Preisig, 2007). Previous, investigations indicate that

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utilization of these types of approaches were capable of accommodating rapidly time-varying sparse channels. However, for underwater acoustic channels with large time spread scale, the huge computational complexity of the two-dimensional parameter optimization poses significant difficulties for the practical applications.

In our previous investigation (Wu and Tong, 2013), a novel p-norm-like-LMS was proposed to incorporate the p-norm like constraint with the LMS algorithm, in which the p parameter was adjusted by gradient descent. Furthermore, in this paper we aimed to exploit the adaptability of the p-norm-like constraint to develop a computationally efficient and effective solution to identify sparse underwater acoustic channels. Specifically, to avoid the huge computational expense caused by the decimal exponential calculation contained in the p-norm-like constraint, we utilized Newton iteration (Richard, 1971; Taylor, 1970) to simplify the gradient descent optimization p-norm-like constraint LMS algorithm (Simplified p-norm-like constraint LMS, S l_p -LMS). Next, we applied it in the estimation of underwater acoustic channels at the presence of time variations. The experimental results obtained with from the simulation, as well as at-sea data verified the effectiveness of the proposed algorithm in improved sparsity exploitation performance and low complexity implementation.

2 Derivation of the simplified *p*-norm like constraint LMS algorithm

2.1 Basis of the norm constraint LMS algorithm

We consider the following minimization problem:

$$\boldsymbol{w}(n) = \arg\min_{\boldsymbol{w}} J_n(\boldsymbol{w}). \tag{1}$$

The cost function $J_n(\boldsymbol{w})$ is defined as:

$$J_n(\boldsymbol{w}) = \left|d(n) - \boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{w}(n)\right|^2 + \gamma\left\|\boldsymbol{w}(n)\right\|_p^p \tag{2}$$

where $d(n)$, $\boldsymbol{w}(n) = [w_0(n), w_2(n), \cdots, w_{L-1}(n)]^{\mathrm{T}}$ and $\boldsymbol{x}(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^{\mathrm{T}}$ denote the desired signal, adaptive filter coefficient vector and input vector respectively, L is the filter length. The estimation square error term $\left|d(n) - \boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{w}(n)\right|^2$ in (2) is commonly used as the cost function in classic LMS-type adaptive methods.

Then, we can see that:

$$\lim_{p\to 0}\left\|\boldsymbol{w}(n)\right\|_p^p = \left\|\boldsymbol{w}(n)\right\|_0 = \#\{i \mid w_i \neq 0\} \tag{3}$$

which means counting the number of nonzero coefficients, and

$$\lim_{p\to 1}\left\|\boldsymbol{w}(n)\right\|_p^p = \left\|\boldsymbol{w}(n)\right\|_1 = \sum_{i=1}^{n}\left|w_i\right| \tag{4}$$

The constraint term as (3) or (4) has been utilized and analyzed popularly for the solution of sparse system estimation to derive various l_0-LMS and l_1-LMS algorithm (Shi and Shi, 2010, 2011). Meanwhile, the cost functions of l_0-LMS and l_1-LMS can be rewritten respectively as:

$$J_{n0}(\boldsymbol{w}) = \left|d(n) - \boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{w}(n)\right|^2 + \gamma\left\|\boldsymbol{w}(n)\right\|_0 \tag{5}$$

$$J_{n1}(\boldsymbol{w}) = \left|d(n) - \boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{w}(n)\right|^2 + \gamma\left\|\boldsymbol{w}(n)\right\|_1 \tag{6}$$

The investigation of Wu and Tong (2013) provides a formal and systematic way to unify the existing norm constraint LMS algorithms into a generalization framework as Eq. (2). Where the last term $\gamma\left\|\boldsymbol{w}(n)\right\|_p^p$ is a p-norm-like constraint term, where, $\gamma > 0$ is a factor to balance the constraint term and the estimation square error, and $\left\|\boldsymbol{w}(n)\right\|_p^p$ is called L_P^P norm or "p-norm-like" (Wu and Tong, 2013).

Thus, the gradient descent recursion of the filter coefficient vector is:

$$\begin{aligned} w_i(n+1) &= w_i(n) - \mu\hat{\nabla}_n = \\ & w_i(n) + \mu e(n)x(n-i) - \\ & \frac{\kappa p\,\mathrm{sgn}[w_i(n)]}{\varepsilon + \left|w_i(n)\right|^{1-p}}, \quad \forall 0 \le i < L \end{aligned} \tag{7}$$

where an element in the vector $\boldsymbol{w}(n)$ is noted as $w_i(n)$, $0 < \varepsilon << 1$ is a constant set for avoiding the ill-condition calculations. μ is the step size parameter of the LMS algorithm, $\kappa = \mu\gamma > 0$ is a parameter combining the contributions of step size and balance factor, $e(n) = d(n) - \boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{w}(n)$ represents estimation error, $\mathrm{sgn}[w_i(n)]$ is the sign function.

The gradient $G_p(n)$ of the diversity measures with respect to the p parameter can be expressed as:

$$G_p(n) = \frac{\partial\left\|\boldsymbol{w}(n)\right\|_p^p}{\partial p} = \left\|\boldsymbol{w}(n)\right\|^p \ln(\left\|\boldsymbol{w}(n)\right\|) \tag{8}$$

With the iteration formula given by:

$$\begin{aligned} p_{n+T} &= p_n - \delta\,\mathrm{sgn}\Big(\frac{1}{T}\sum_{j=n}^{n+T} G_p(j)\Big) \\ &= p_n - \delta\,\mathrm{sgn}\Big\{\frac{1}{T}\sum_{j=n}^{n+T}[w_j(n)] - 1\Big\}, \end{aligned} \tag{9}$$

where δ is a constant factor to control the step size of descent gradient updating.

2.2 The simplification of the p-norm like constraint algorithm

Considering the practical algorithm implementation in engineering applications, the decimal exponential computation contained in the proposed algorithm will lead to significant complexity. The simplification of the proposed p-norm like constraint algorithm has been addressed in this paper.

From the formula (7) and (9), we can see that the main problem of simplification is to solve $\left|w_i(n)\right|^{1-p_n}$ and to avoid the use of decimal exponential calculation. Devoid of loss of generality, we set $\delta = \frac{1}{\Delta}$ where Δ as a positive integer adopted to avoid the use of decimal. Thus, the term 1-p_n can be expressed as $m \cdot \frac{1}{\Delta}$, where $m = \Delta \cdot (1-p_n)$ and is a positive integer between 0 and Δ. Next, we have:

$$\left|w_i(n)\right|^{1-p_n} = \left|w_i(n)\right|^{\frac{m}{\Delta}} \tag{10}$$

For convenience, we set $g = \left|w_i(n)\right|^{\frac{m}{\Delta}}$, and it can be rewritten as:

$$g^{\Delta} - \left|w_i(n)\right|^m = 0 \tag{11}$$

With Newton iterative method (Richard, 1971; Taylor, 1970), we have initial iteration as:

$$g_2 = g_1 - \frac{(g^{\Delta} - \left|w_i(n)\right|^m)}{(g^{\Delta} - \left|w_i(n)\right|^m)'}\Big|_{g=g_1} \tag{12}$$

Here, $(g^{\Delta} - \left|w_i(n)\right|^m)'$ means a derivation of g. Then the iteration of it can be written as:

$$\begin{aligned} g_{j+1} &= g_j - \frac{(g_j^{\Delta} - \left|w_i(n)\right|^m)}{\Delta * g_j^{\Delta-1}} \\ &= \frac{\Delta-1}{\Delta} g_j + \frac{\left|w_i(n)\right|^{\Delta*(1-p_n)}}{\Delta * g_j^{\Delta-1}} \end{aligned} \tag{13}$$

It has been recognized that Newton's method converges much faster towards a local maximum or minimum than the gradient descent method does (Richard, 1971; Taylor, 1970). Hence, the number of iterations can be a small number like 3 or 4 and still meet the general performance requirement.

A detailed description of the proposed algorithm is described using MATLAB pseudo-codes as follow:

Given $\mu, \kappa, p, \varepsilon, L, T$

Initial $\boldsymbol{w} = \text{zeros}(L,\ 1), p_{initial}$

for $n = 1, 2\cdots$

Input new $\boldsymbol{x}(n)$ and $d(n)$;

$e(n) = d(n) - \boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{w}(n)$;

$p_{n+1} = p_n - \frac{1}{\Delta} * \text{sgn}\{\frac{1}{T}\sum_{j=n}^{n+T}[w_j(n)] - 1\}$;

$g_1 = 1$;

for $j = 1:3$

$g_{j+1} = \frac{\Delta-1}{\Delta} g_j + \frac{\left|w_i(n)\right|^{\Delta*(1-p_n)}}{\Delta * g_j^{\Delta-1}}, \quad \forall 0 \le i < L;$

end

$f(n) = p_{n+1}\text{sign}(w_i(n)) \cdot /(\varepsilon + \text{abs}(g_{j+1}))$;

$w_i(n+1) = w_i(n) + \mu e(n) x(n-i) - \kappa f(n), \quad \forall 0 \le i < L$

end

Compared to the classic LMS algorithm and norm constraint LMS algorithms (Wu and Tong, 2013), the computational complexity per iteration of the simplified p-norm-like algorithms is listed as Table 1. As the Table 1 indicating, by approximating the exponential calculation with the Newton iteration method, the proposed algorithm considerably alleviates the computational complexity of the p-norm-like constraint LMS algorithm. In each iteration, l_p -LMS has L+1 exponential calculation and the exponential calculation is implemented by the form of table look-at for considering its complexity (Wu and Tong, 2013). However, this process is avoided in Sl_p-LMS with the Newton iteration method, so, it to some extent converted amount of computation to addition and multiplication in each iteration.

Table 1 Computational complexity per iteration of each algorithm

Algorithms	Addition	Multiplication	Sign	Exponential calculation
LMS	2L	2L+1	NA	NA
l_0 - LMS	3L	2L	NA	NA
l_p - LMS	3L+1	2L+1	L	L+1
Sl_p-LMS	3L+4	5L+7	L	NA

3 Numerical simulations

In this section the performance of simplified p-norm-like constraint LMS (S l_p-LMS) was compared with that of the standard LMS algorithm, l_0-norm penalty constraint LMS and p-norm-like constraint LMS (l_p -LMS) (Shi and Shi, 2010, 2011; Wu and Tong, 2013) to evaluate the tradeoff between the channel estimation performance and the algorithm complexity. Firstly, the numerical simulation was designed to analyze the performance of the proposed algorithms. For the algorithm comparison of channel estimation, two performance metrics were used (Li and Preisig, 2007): channel prediction error $\boldsymbol{\varepsilon}_{\boldsymbol{w}}^2$ and signal residual prediction error $\boldsymbol{\varepsilon}_{\boldsymbol{y}}^2$, which are defined as Eq. (14) and Eq.(15) respectively:

$$\boldsymbol{\varepsilon}_{\boldsymbol{w}}^2 = [w_i(n) - \tilde{w}_i(n)]^2, \quad \forall 0 \le i < L,\ \ n = 1, 2, \ldots \tag{14}$$

$$\boldsymbol{\varepsilon}_{\boldsymbol{y}}^2 = [\tilde{y}_i(n+1) - \tilde{y}_i(n)]^2, \quad \forall 0 \le i < L,\ \ n = 1, 2, \ldots \tag{15}$$

where $\tilde{w}_i(n)$ is the estimation of the channel impulse response at n time, $w_i(n)$ is the true one, and $\tilde{y}_i(n)$ is the estimation of the channel output signal at n time. Although the channel prediction error $\boldsymbol{\varepsilon}_{\boldsymbol{w}}^2$ is often used in simulations, it is not a very useful metric in practice because in reality the true channel impulse response is usually unknown. Instead, the signal residual prediction error $\boldsymbol{\varepsilon}_{\boldsymbol{y}}^2$ is often used as a good surrogate for $\boldsymbol{\varepsilon}_{\boldsymbol{w}}^2$. The mean square of errors (MSE) $\boldsymbol{\varepsilon}_{\boldsymbol{w}}^2$ and $\boldsymbol{\varepsilon}_{\boldsymbol{y}}^2$ will be used for the remainder of this paper to evaluate various channel estimation algorithms.

In the numerical simulation, similar to the prior investigations (Stojanovic, 2005; Hosein, 2009), sparse channels are artificially created using mixture of Gaussian (MOG) model, which is also called the Bernoulli-Gaussian model:

$$w_i \sim a \cdot N(0,\sigma_M) + (1-a) \cdot N(0,\sigma_m), \quad \forall 0 \le i < L \quad (16)$$

where a denotes probability of activity of the larger taps, σ_M and σ_m are the standard deviations of the larger taps and smaller ones, respectively. A typically sparse system with 50 coefficients was adopted. We initially set the 6th and 7th tap as dominant coefficients, associated with magnitude of 1.0 and −0.5 respectively, with all the other coefficients set to zero. The signal adopted to excite the sparse system was white Gaussian random sequences with the length of signal N=4000, zero mean, and standard unit deviation. The output of the sparse system was mixed with the white Gaussian random noise to create an SNR of 40 dB. We adopted Sparsity Ratio (SR) to quantitatively evaluate the sparsity of unknown system, which has been defined as the ratio of the number of non-zero taps to the total number of coefficients. All the algorithms were simulated for 100 times.

To simulate the variation of the channel characteristics, during the simulation the SR of the sparse system varies only once, taking place at the 2000th iteration. After variation, four new dominant coefficients were added to produce two different SRs 2/50, 6/50 of the target sparse system corresponding to the initial 2000 iterations and the second 2000 iterations. The locations of the newly added dominant coefficients were chosen according to Gaussian distribution within the range of global impulse response. The magnitudes of the newly added dominant coefficients were created with Gaussian random value between −1 and 1. The length of filter L was 50 for all the selected algorithms. The parameters of all the algorithms were carefully chosen to guarantee the same rate of convergence to facilitate the comparison of the steady-state errors, as listed in Table 2. For the proposed algorithm, the initial p value was empirically set to 1.

Table 2 Parameters of candidate algorithms

Algorithms	μ	κ	δ	T
LMS	0.02	NA	NA	NA
l_0 - LMS	0.02	1.0e-4	NA	NA
Sl_p-LMS	*0.02*	*1.0e-4*	*0.05*	*10*
l_p - LMS	0.02	1.0e-4	0.01	10

As shown in Fig. 1, the proposed S l_p- LMS yields better steady-state error of ε_w^2 than the standard LMS or l_0-norm LMS does, while the classic l_p-LMS achieves the smallest MSE than all the other methods. It indicates that, the proposed simplified algorithm can attain low complexity implementation at the expense of a slight sacrifice of steady-state error performance. In view of the practical cases where the real channel is unknown, we also evaluated candidate algorithms in numerical simulations with MSEs of ε_y^2, as defined in Eq. (18). The Fig. 2 provides the performance curves of the proposed and referenced algorithms. From Fig. 2, one can see that, while the qualitative result of the performance comparison is the same, the quantitative performance gap of the proposed algorithm with respect to other algorithms was reduced in terms of the steady-state error of ε_y^2.

Shown in Fig. 3 are the iteration curves of the p parameter under the classic and the proposed simplified p-norm like constraint LMS algorithm, from which we can see that the behaviors of p optimization of S l_p-LMS and l_p-LMS algorithm exhibit similar pattern with the variations of the SR. Meanwhile, there exist small-range discrete fluctuations of p parameter at the iteration curve of the proposed S l_p-LMS algorithm, which is caused by the adoption of the Newton iteration to replace the decimal exponential calculation.

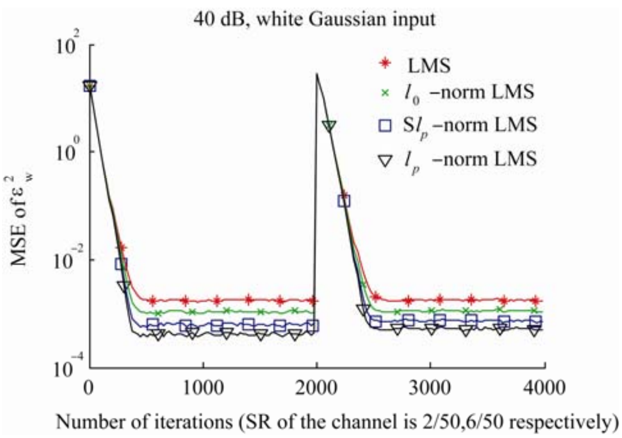

Fig. 1 MSEs of ε_w^2 of different algorithms for the sparse channel

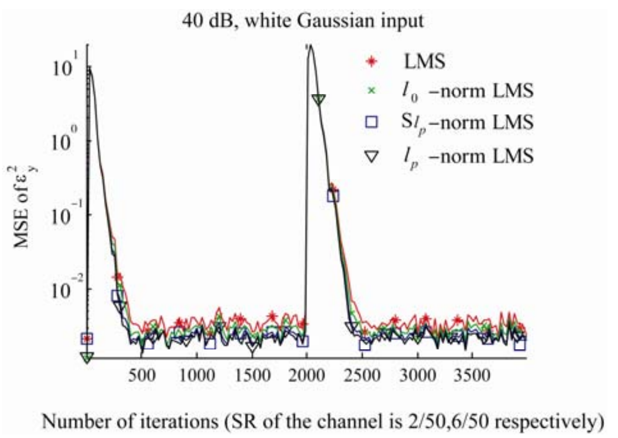

Fig. 2 MSEs of ε_y^2 of different algorithms for the sparse channel

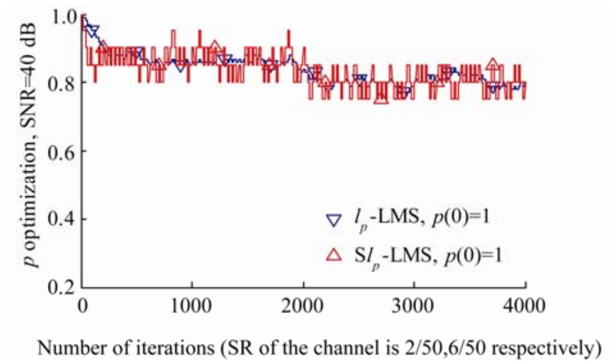

Fig. 3 Iterative optimization of p parameter for the simulated channel

4 Design of at-sea experiment

The coherent UWA communication link used to test the algorithm performance is shown in Fig. 4. The link consists of two computers (one acting as a bit resource while the other acting as a signal recorder): a DA and AD card used for data output and acquisition, power amplifier, preamplifier, and two transducers (one for transmitting and the other for receiving).

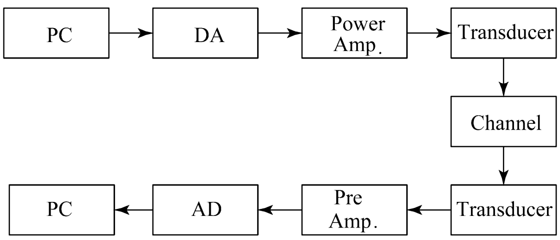

Fig. 4 Bock chart of experimental UWA link

The signal frame consisted of a linear frequency modulation chirp to acquire synchronization and detect the channel, a guard time and the modulated data. The carrier frequency was 16k Hz, with the sampling rate at 96ksps. The modulation format was BPSK, and the signals were transmitted at 6.4 kilobits per second. The bandwidth of the transducer couple was 13-18 kHz.

The experiment in the ocean was conducted at Wuyuan Bay, Xiamen, China. The depth of the experiment area was approximately 7m under the pier and 12 m offshore. The transmit transducer was suspended from the pier at the depth of 5 m from the sea surface. Similarly, the receive transducer was suspended from a boat at the depth of 5 m from the sea surface. The distance between the transmitter and receiver was 2 km.

The adaptive channel estimation algorithm described in the previous section was implemented in MATLAB and used for off-line processing of experimental data. For the purpose of comparison, classic LMS, l_0-LMS, S l_p-LMS and l_p-LMS algorithms were also adopted to process the signal. The parameters of the proposed and reference algorithms were chosen to minimize the corresponding bit error rate, as shown in Table.3. The multipath intensity profile (MIP) of the experimental channel obtained with the matched filtering of the periodic linear frequency modulation (LFM) pulses is shown in Fig. 5. It can be observed that the experimental underwater acoustic channel exhibits typical multipath structure, sparseness and time variations. The raw data received signal recorded during the sea experiment had a signal noise ratio (SNR) of 14 dB.

Table 3 Parameters of candidate algorithms for experimental data

Algorithms	μ	κ	δ	T
LMS	0.01	NA	NA	NA
l_0-LMS	0.01	10^{-4}	NA	NA
Sl_p-LMS	0.01	10^{-4}	0.05	10
l_p-LMS	0.01	10^{-4}	0.01	10

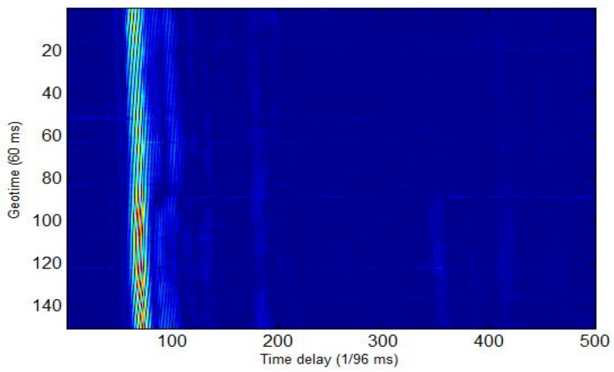

Fig. 5 Multipath intensity profile of the experimental underwater channel

As the exact physical channel impulse response is unknown in the practical at-sea experiment, the signal residual prediction error ε_y^2 instead of ε_w^2 is adopted for the performance comparison. The MSE results of the different algorithms are shown in Fig.6. As we can observe from Fig.6, l_0-LMS yields better MSE than LMS, l_p -LMS yields better MSE than l_0-LMS, which is consistent with previous investigation (Gu *et al*., 2009; Jin *et al*., 2010; Shi and Shi, 2010, 2011; Wu and Tong, 2013). After the algorithm simplification, the proposed algorithm achieved a slightly lower performance than the classic l_p-LMS did, indicating a good agreement with the result of numerical simulation. Similar to the numerical simulation, from Fig.7, we can see that the gradient descent p optimization of S l_p - LMS and l_p - LMS generally exhibit the same variation pattern, with the proposed simplification of the S l_p - LMS algorithm producing a small-range discrete fluctuation effect on the iteration of p parameter.

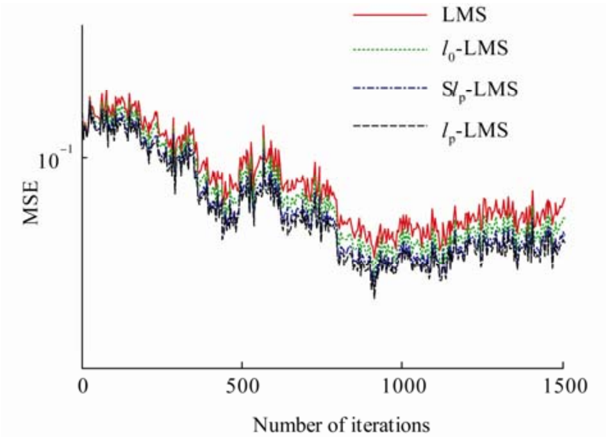

Fig. 6 MSEs of different algorithms for the experimental underwater channel

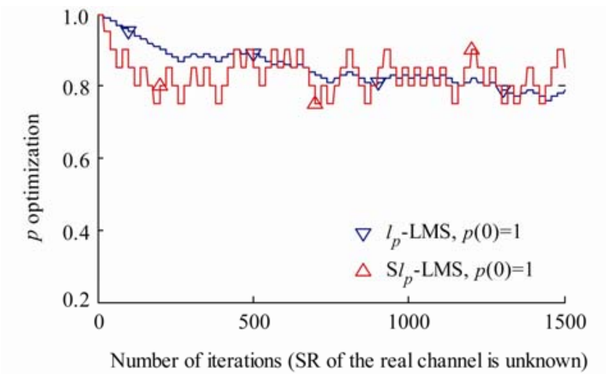

Fig. 7 Iterative optimization *of p* parameter for the experimental channel

Furthermore, as the purpose of the channel estimation is to provide the channel response to construct channel equalizer, the accuracy of the channel estimation will determine the performance of the equalizer, as well as the communication quality. Thus, the channel responses obtained in the experiment with the proposed and reference algorithms are used to construct a linear minimum mean-square error (LMMSE) equalizer to achieve symbol recovery, the bit error rate (BER) of which is adopted to evaluate the performance of those algorithms. The input-output relationship of the LMMSE equalization can be written as:

$$\hat{s} = (\boldsymbol{H}^H \boldsymbol{H} + \sigma_w^2 \boldsymbol{I})^{-1} \boldsymbol{H}^H x \qquad (16)$$

where $\boldsymbol{I}$ denotes the identity matrix and $\boldsymbol{H}$ is the underwater acoustic channel response obtained with channel estimation algorithm. So the LMMSE equalizer is a channel estimate-based equalizer and thus the associated performance is closely related to the channel estimation accuracy (Zeng and Xu, 2012).

The length of the LMMSE equalizer is 50, the same as that of the channel estimator. As the experimental underwater channel exhibits typical time variations as shown in Fig.5, the LMMSE equalizer is calculated every 50 symbols with the channel estimation result to accommodate the time variations.

The BER obtained by the LMMSE equalizer calculated with the LMS, l_0-LMS, l_p-LMS and the proposed simplified l_p-LMS algorithm is 9.78%, 6.55%, 5.70%, and 6.27% respectively. It is evident that the BER performance of the channel estimation based LMMSE equalizer as well validates the effectiveness of the proposed simplified algorithm, which achieving a BER worse than that of the l_p-MS, but better than that of LMS and l_0-LMS.

5 Conclusions

In order to improve the performance of sparse system identification at the expense of low computational complexity, a new simplified algorithm was derived in this paper, through incorporating p-norm-like constraint with classic LMS algorithm, and lastly utilizing Newton iteration algorithm to simplify the algorithm implementation process. With the proposed approximation of the decimal exponential calculation, the computational complexity can be significantly reduced at the expense of a slight sacrifice of performance. Numerical simulation and experimental results demonstrate the effectiveness of the proposed simplified l_p-LMS in sparsity exploitation performance enhancement and computational complexity saving compared to the classic norm constraint LMS algorithms. From a practical point of view, the proposed low-complexity l_p-LMS algorithm has the potential of being applied in the practical identification of sparse underwater acoustic channels.

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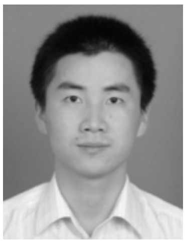

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