Density-Spicity coordinates
&
the applications

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New definition of potential spicity by the least square method
Rui Xin Huang, Lu-Sha Yu, Sheng-Qi Zhou
JGR, Ocean

Heaving, Stretching, spicing modes: climate variability in the ocean
Rui Xin Huang
Springer & Higher Education Press of China (in press)

Isopycnal analysis --- the backbone of physical oceanography

1) Large-scale circulation (1900-2000): density is the primary controller:

vertical distribution→ stratification

horizontal distribution → pressure gradient → current

- On isopycnal surfaces: variability is in spicity

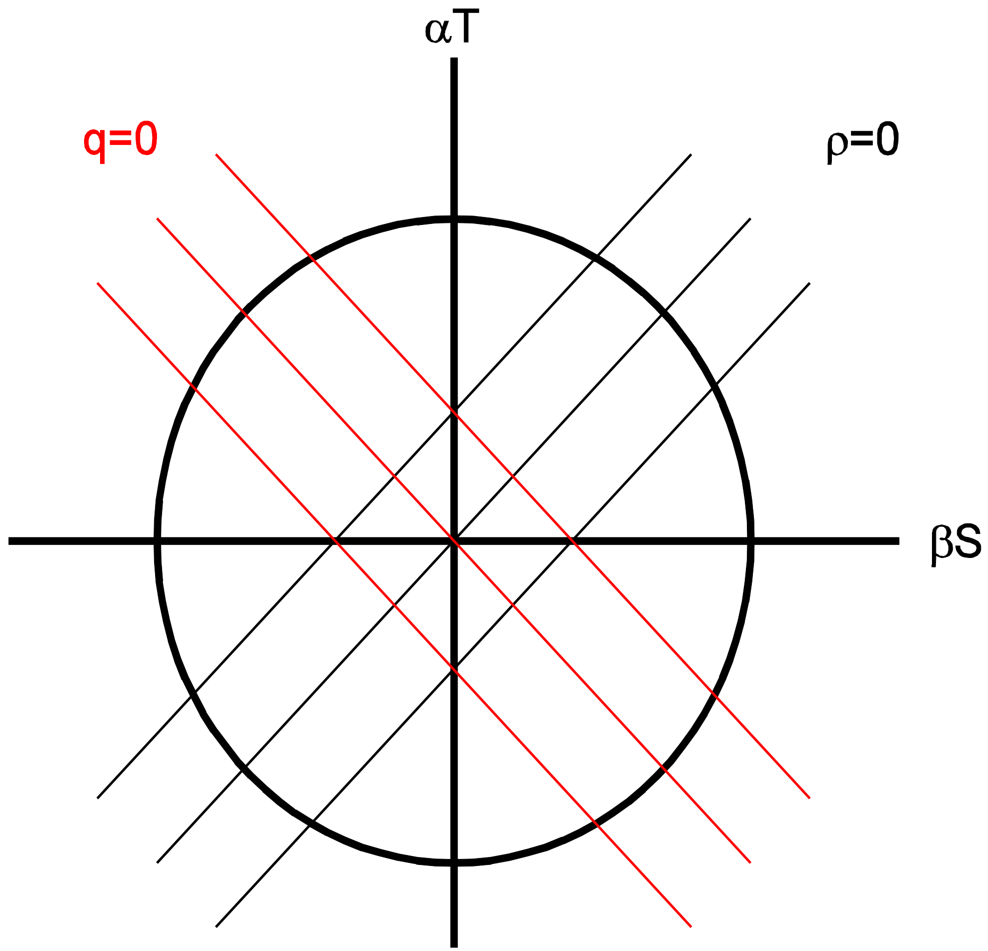
2) Small scale circulation (2001-?): spicity is the dominating player

- Vertical distribution→ double diffusion
- Horizontal gradient → lateral mixing ??
- On isopycnal surface: spicity spectrum is critical

A 55-year old puzzle

Stommel (1962): "On the cause of the temperature-salinity curve in the ocean":

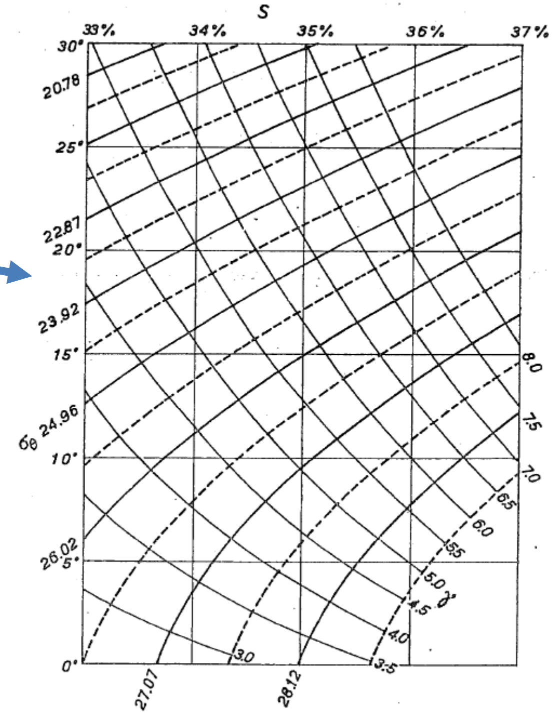
On an isopycnal surface density is constant, another thermodynamic function orthogonal to density is desirable…



The truth or fault??

1) Veronis (1972); Mamayev (1975)

Got the nearly orthogonal function



2) The orthogonality constraint was abandoned

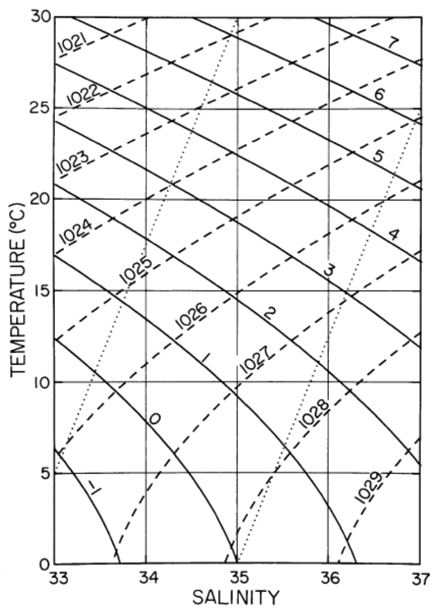
Jackett, D. R. and T. J. McDougall (1985)

Flament (2002), McDougall and Krzysik (2015):

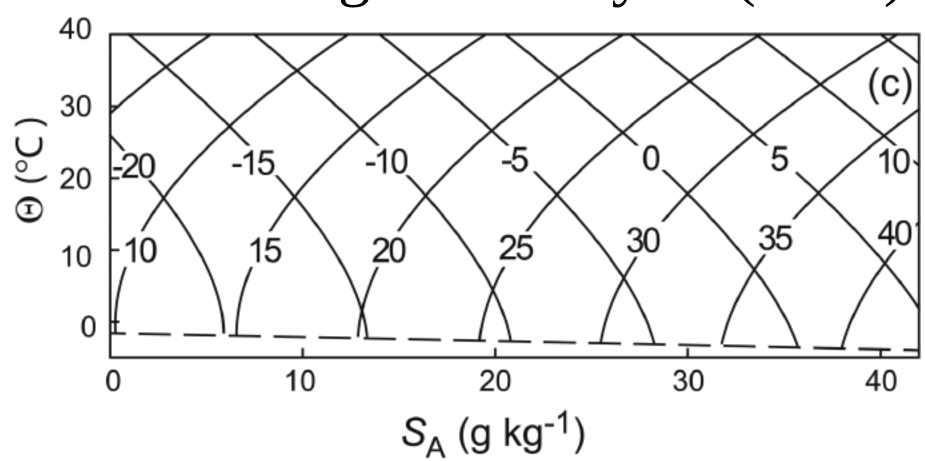
"It is impossible to construct orthogonal functions;

The orthogonality has no physical meaning"

Flament (2002)



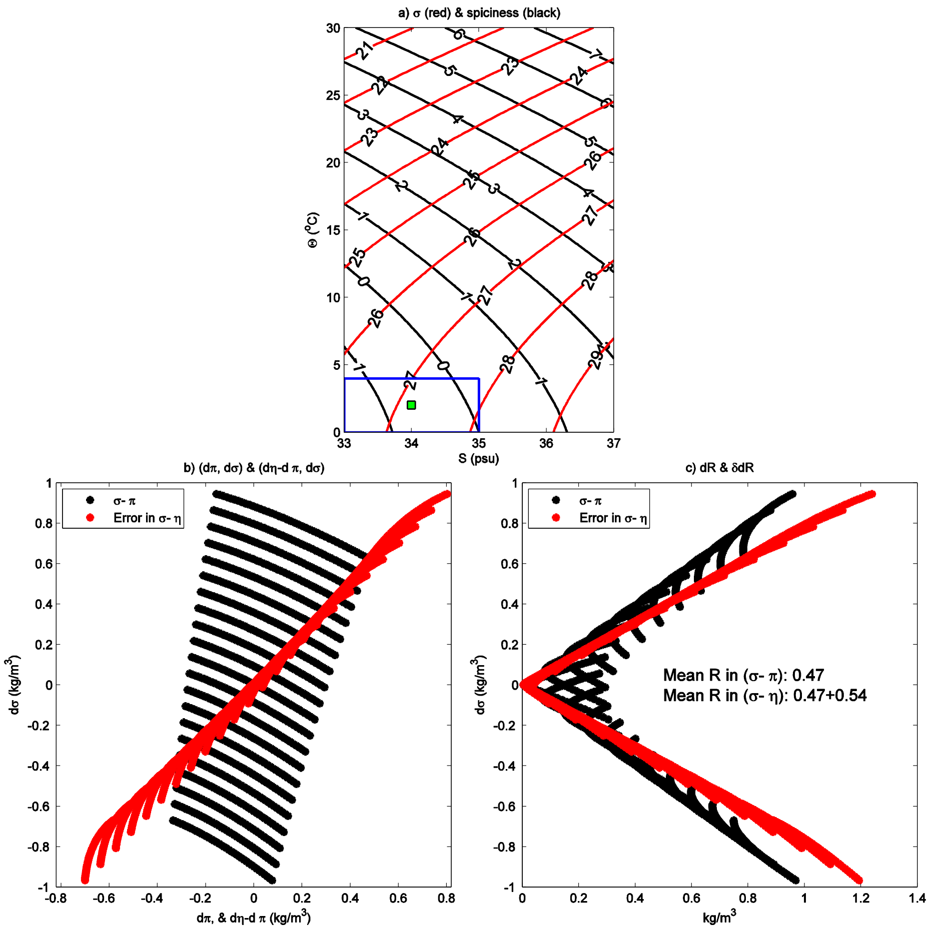
McDougall & Krzysik (2015)

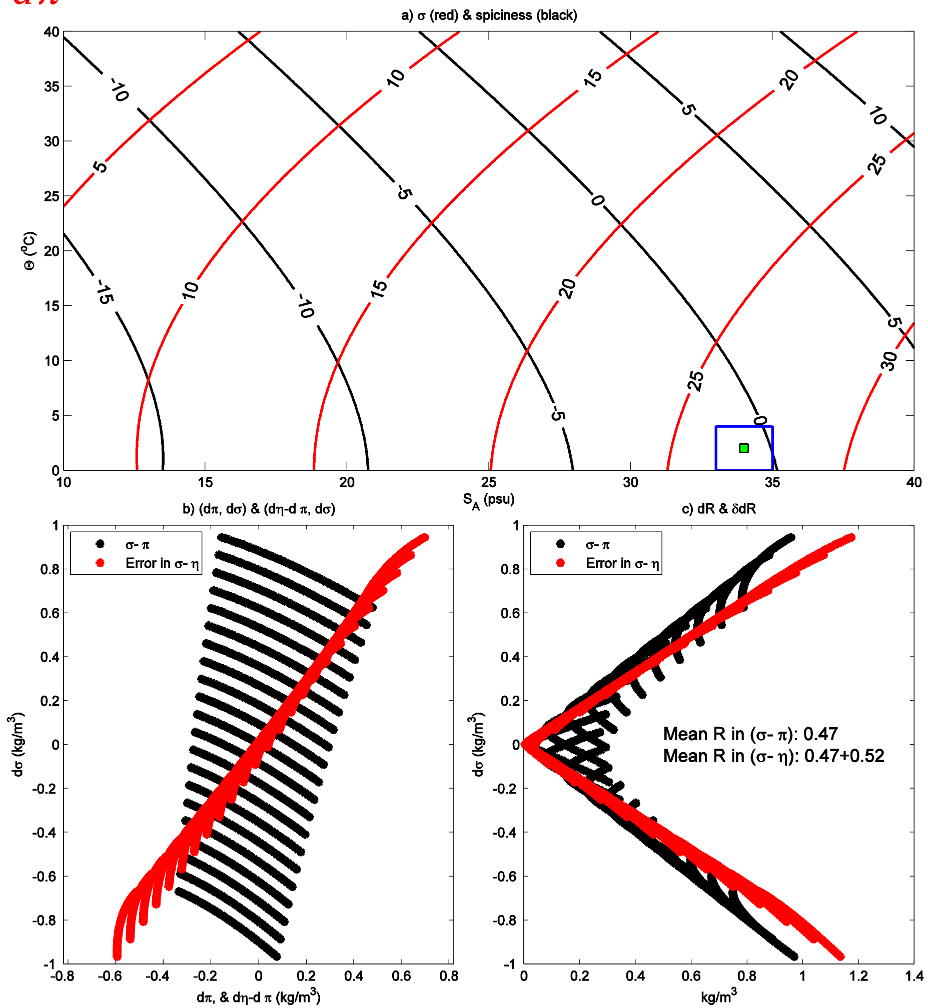


Density & spiciness contours are nearly parallel at low temperature → they are poor choices for coordinate system

Orthogonal coordinates can present climate signals most concisely

Radius of signal $R = \sqrt{d\sigma^2 + d\pi^2}$





Searching for spicity

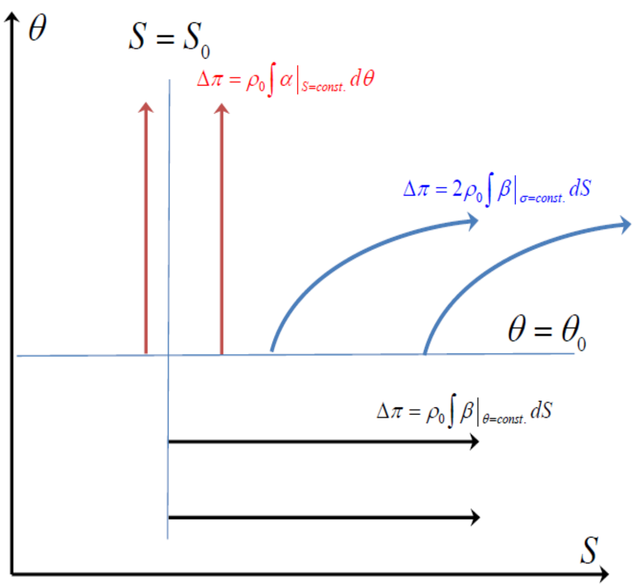
$$\boxed{\nabla\sigma = \rho_0\left(-\alpha\nabla\theta + \beta\nabla S\right)}$$

The incorrect choice:

$$\nabla\pi = \rho_0(\alpha\nabla\theta + \beta\nabla S)$$

Flipping the sign does not make them orthogonal

McDougall and Rrzysik (2015): gave up the orthogonality, and define spicity by path integral



Along σ contours:

$$\Delta\sigma = 0 \rightarrow \alpha\Delta\theta = \beta\Delta S$$

$$\rightarrow \Delta\pi = 2\beta\Delta S$$

$$\Delta\pi = 2\rho_0 \int_{S_0}^{S} \beta\big|_{\sigma=const.}\, dS$$

The correct choice:

$$\boxed{\nabla\pi = \rho_0 c\left(\theta, S\right)(\beta\nabla\theta + \alpha\nabla S)}$$

Search for a scalar function which gradient vector is orthogonal to a given vector field

- For a given vector, $\vec{g} = \left(g^x, g^y\right)$, if $\partial g^y / \partial x = \partial g^x / \partial y$, it is called a conserved field; for such a vector we can find out a scalar function H(x,y) by integration along a path, and $\nabla H(x,y) = \vec{g}$
- Vector fields often do not satisfy such constraint
- We can define H in the least square sense
- For any given function F, we can use 2-step linearization to find a function which contours are orthogonal to F in the least square sense.

Potential spicity defined in the least square sense

$$\Delta\varepsilon = RMS\left[\arcsin\left(\frac{\nabla\sigma\cdot\nabla\pi}{|\nabla\sigma\|\nabla\pi|}\right)\right] = Minimum$$

- # Step One:

Find a vector field: the lengths of each component match those of the original vector in the least-square sense

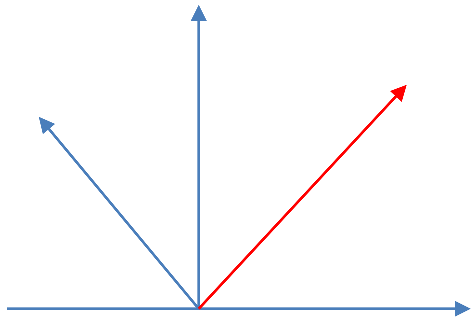
Fitting the solution by a polynomial.

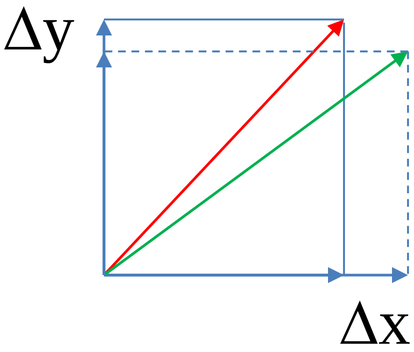
$$\Delta\varepsilon = \sum\left[\left(\sigma^x - \pi^y\right)^2 + \left(\sigma^y + \pi^x\right)^2\right] = Minimum$$

Thus, the solution is defined over the whole domain, not along each integral pathway

Search for a vector

Rotating 90^o to create the target vector → Search for a testing vector → Solving a linear problem





$$D = \Sigma\left[(\Delta x)^2 + (\Delta y)^2\right] = \min$$

Formulating the Least square problem

A 4th-order polynomial for fitting (10th-order polynomial is actually used in calculation)

$$f = a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 + a_6 x^3 + a_7 x^2 y + a_8 xy^2 + a_9 y^3 + a_{10} x^4 + a_{11} x^3 y + a_{12} x^2 y^2 + a_{13} xy^3 + a_{14} y^4$$

- The target function

$$D = \sum_{i,j} \left[\left(\Delta f_{i,j}^{x} - X_{i,j} \right)^2 + \left(\Delta f_{i,j}^{y} - Y_{i,j} \right)^2 \right]$$

- The constraint of LSP

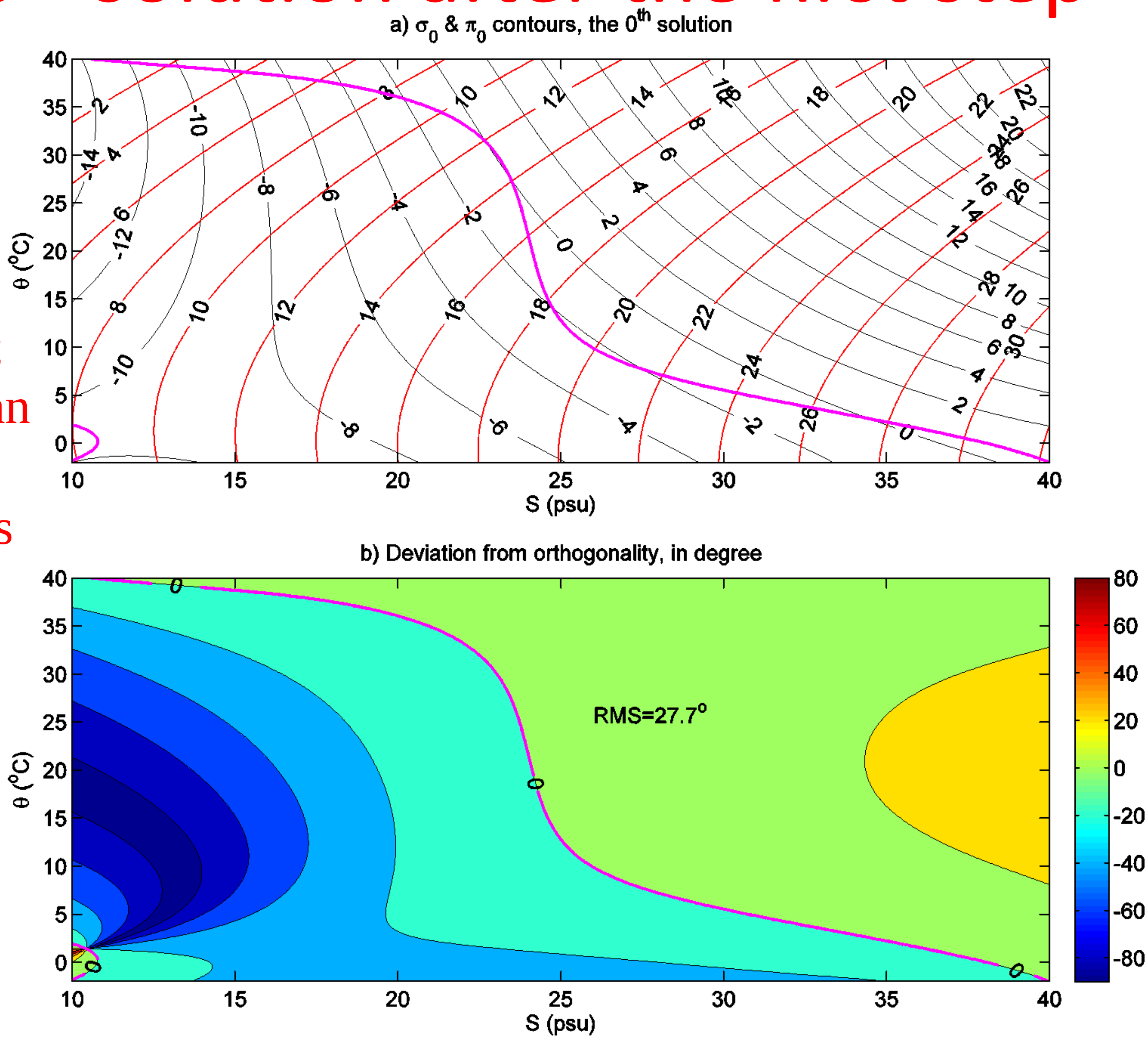
$$\delta D / \delta a_k = 0, k = 1, 2, 3, \ldots, 12, 13, 14$$

- The searching result

$$F \cdot \vec{a} = \vec{d} \rightarrow \vec{a} = F^{-1} \vec{d}$$

The 0th solution after the first step

It is not good; however, it can be improved with iterations in the second step



Step Two:

$$\Delta\lambda = RMS\left(\frac{\nabla\sigma\cdot\nabla\pi}{|\nabla\sigma\|\nabla\pi|}\right) = Minimum$$

Linearization → $\Delta\lambda = RMS\left(\frac{\nabla\sigma\cdot\nabla\pi}{|\nabla\sigma\|\nabla\pi_0|}\right) = Minimum$

$$\delta C/\delta a_k = 0, k = 1,2,3,...,14$$

$$\begin{bmatrix} b_{1,1} & b_{1,2} & . & b_{1,14} \\ b_{2,1} & b_{2,2} & . & b_{2,14} \\ . & . & . & . \\ b_{14,1} & b_{14,2} & . & b_{14,14} \end{bmatrix}\begin{pmatrix} a_1 \\ a_2 \\ . \\ a_{14} \end{pmatrix} = 0$$

This system has a trivial solution only $a_k \equiv 0, k = 1,2,3,...,14$

For a whole year it was unclear how to find a meaningful solution; however, never give up !!

Separating the system into 2 parts

$$\begin{bmatrix} \boxed{b_{1,1}^{0} & b_{1,2}^{0} & . & b_{1,14}^{0}} \\ b_{2,1}^{0} & b_{2,2}^{0} & . & b_{2,14}^{0} \\ . & . & . & . \\ b_{14,1}^{0} & b_{14,2}^{0} & . & b_{14,14}^{0} \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ . \\ a_{14} \end{pmatrix} = 0$$

Using 0th solution to find the 1st iteration solution

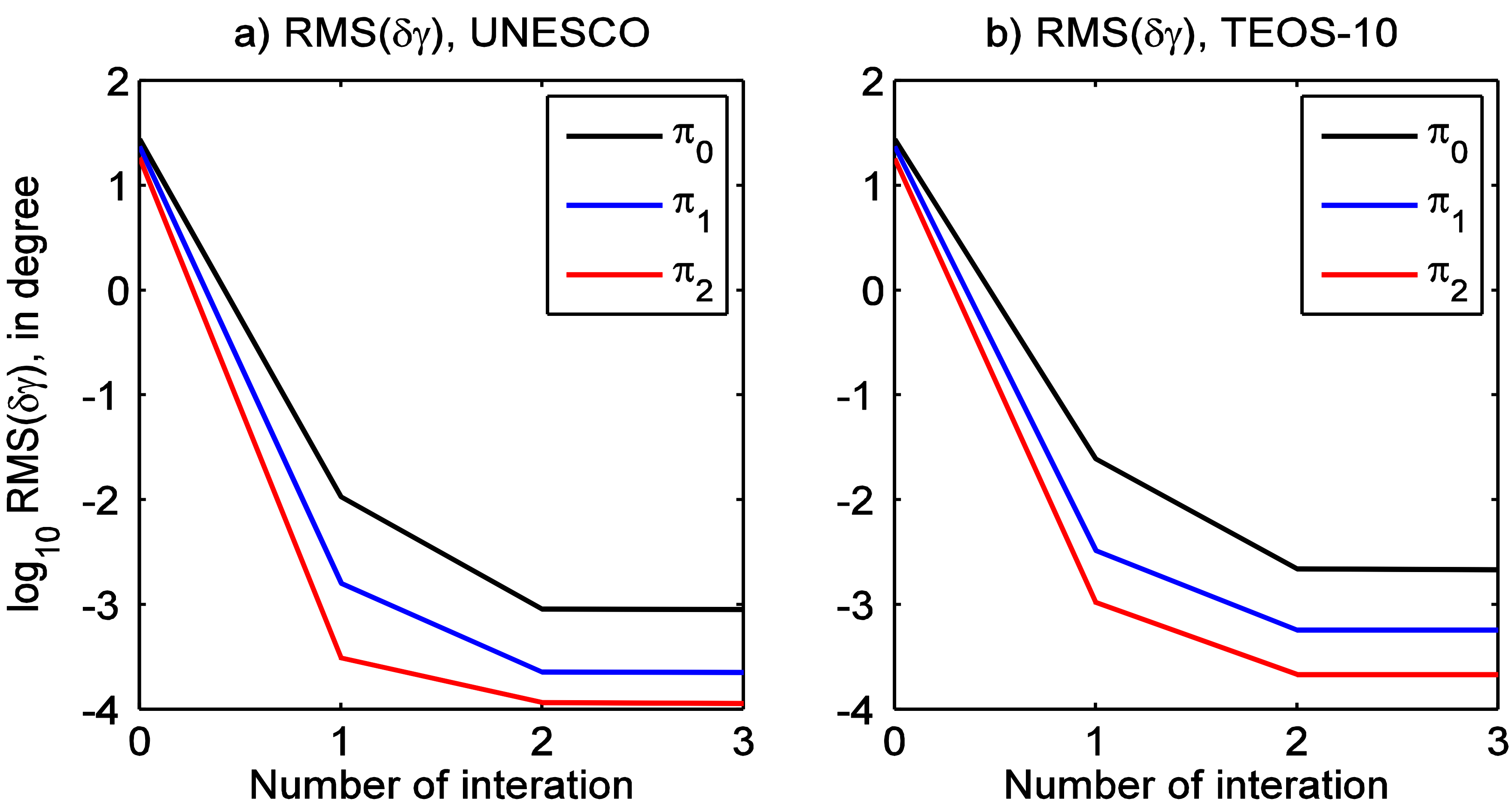
$$b_{1,1}^{0} a_1^{1} = -(b_{1,2}^{0} a_2^{0} + b_{1,3}^{0} a_3^{0} + \ldots + b_{1,14}^{0} a_{14}^{0})$$

$$\begin{bmatrix} b_{2,2}^{0} & b_{2,3}^{0} & . & b_{2,14}^{0} \\ b_{3,2}^{0} & b_{3,3}^{0} & . & b_{3,14}^{0} \\ . & . & . & . \\ b_{14,2}^{0} & b_{14,3}^{0} & . & b_{14,14}^{0} \end{bmatrix} \begin{pmatrix} a_2^{1} \\ a_3^{1} \\ . \\ a_{14}^{1} \end{pmatrix} = -a_1^{1} \begin{pmatrix} b_{2,1}^{0} \\ b_{3,1}^{0} \\ . \\ b_{14,1}^{0} \end{pmatrix}$$

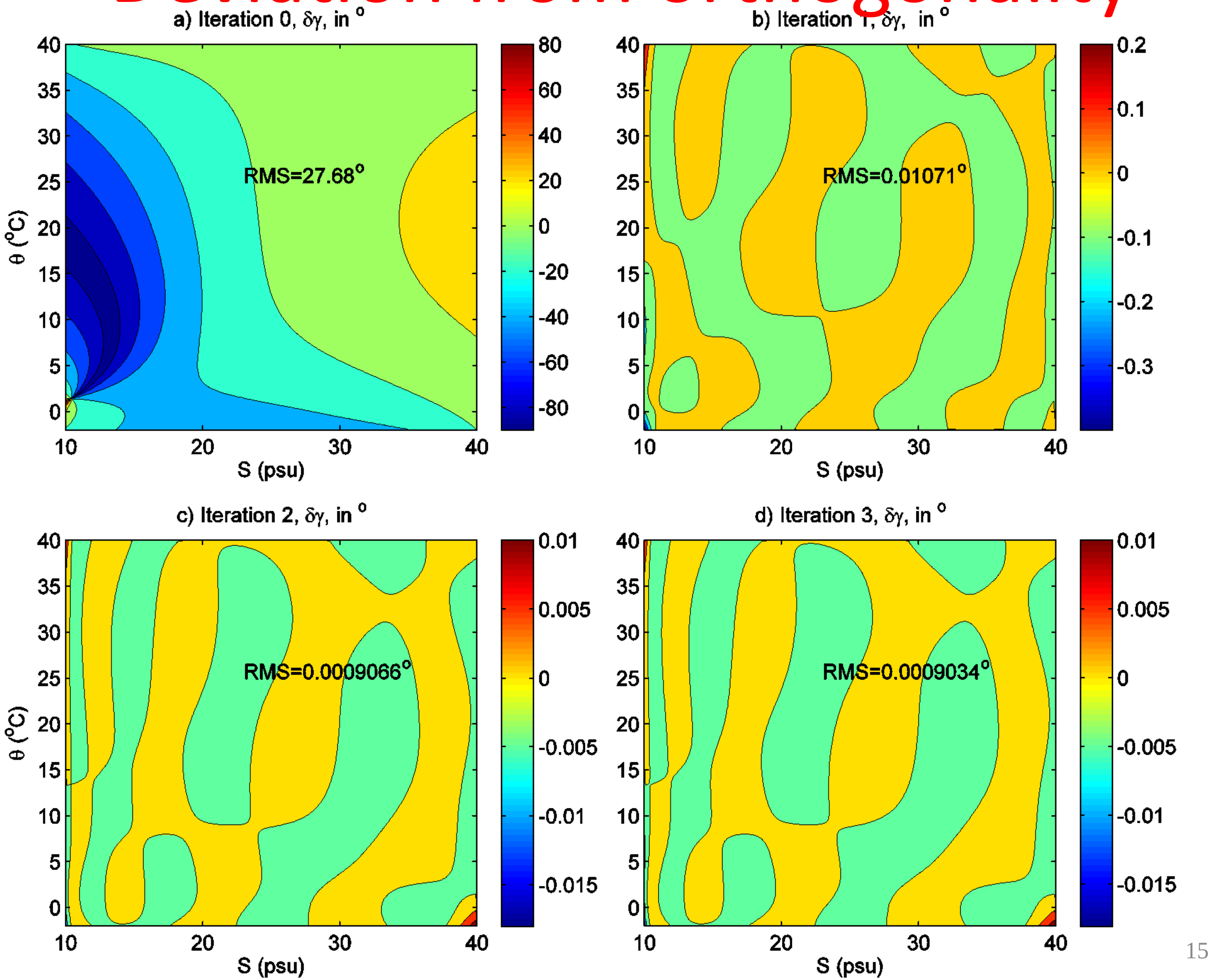
The RMS (angle errors) declines rapidly (Based on 10th-order polynomial)

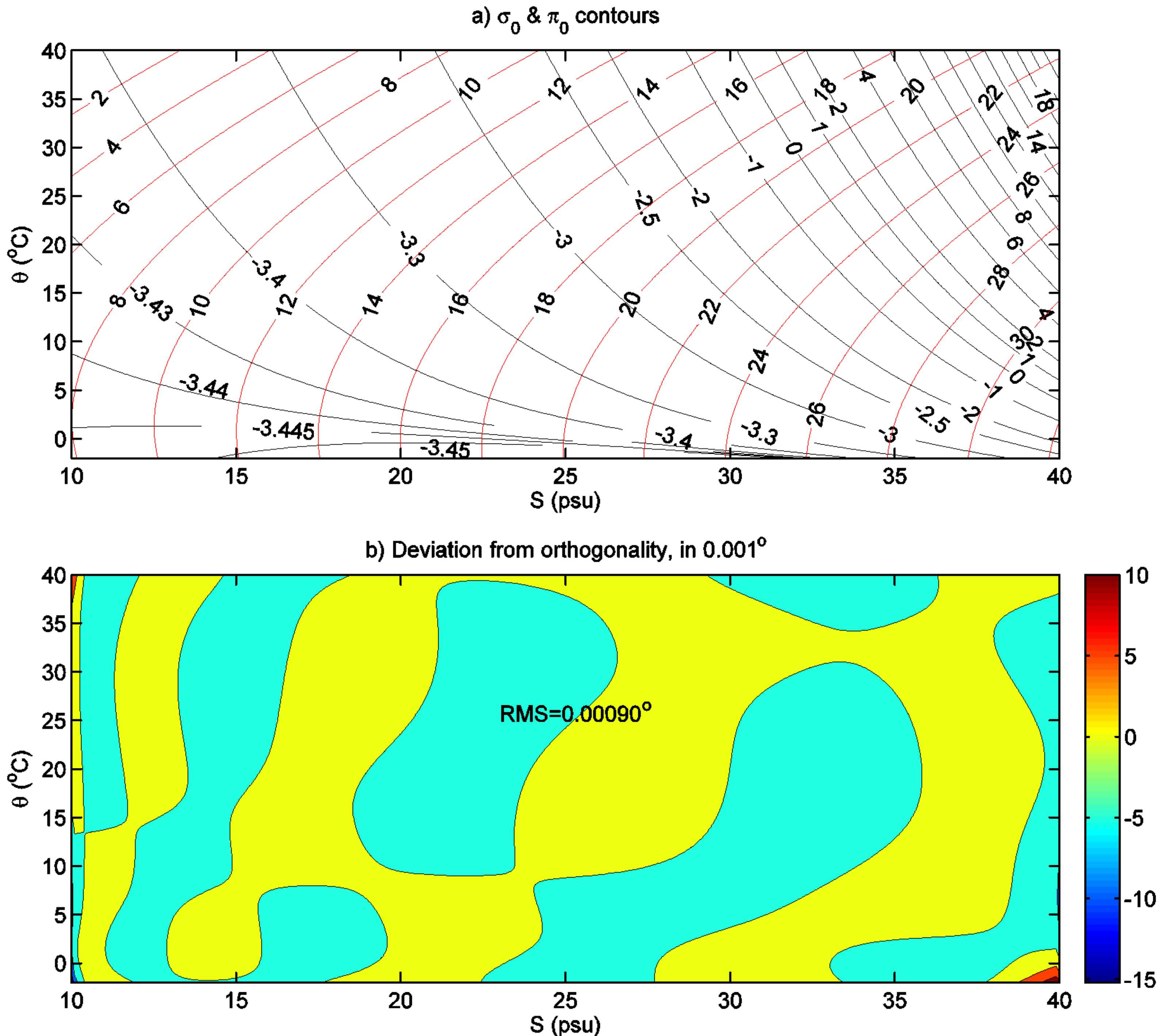
0 – initial result from the first step of linearization

1,2,3 – results from the second step of linearization



Deviation from orthogonality

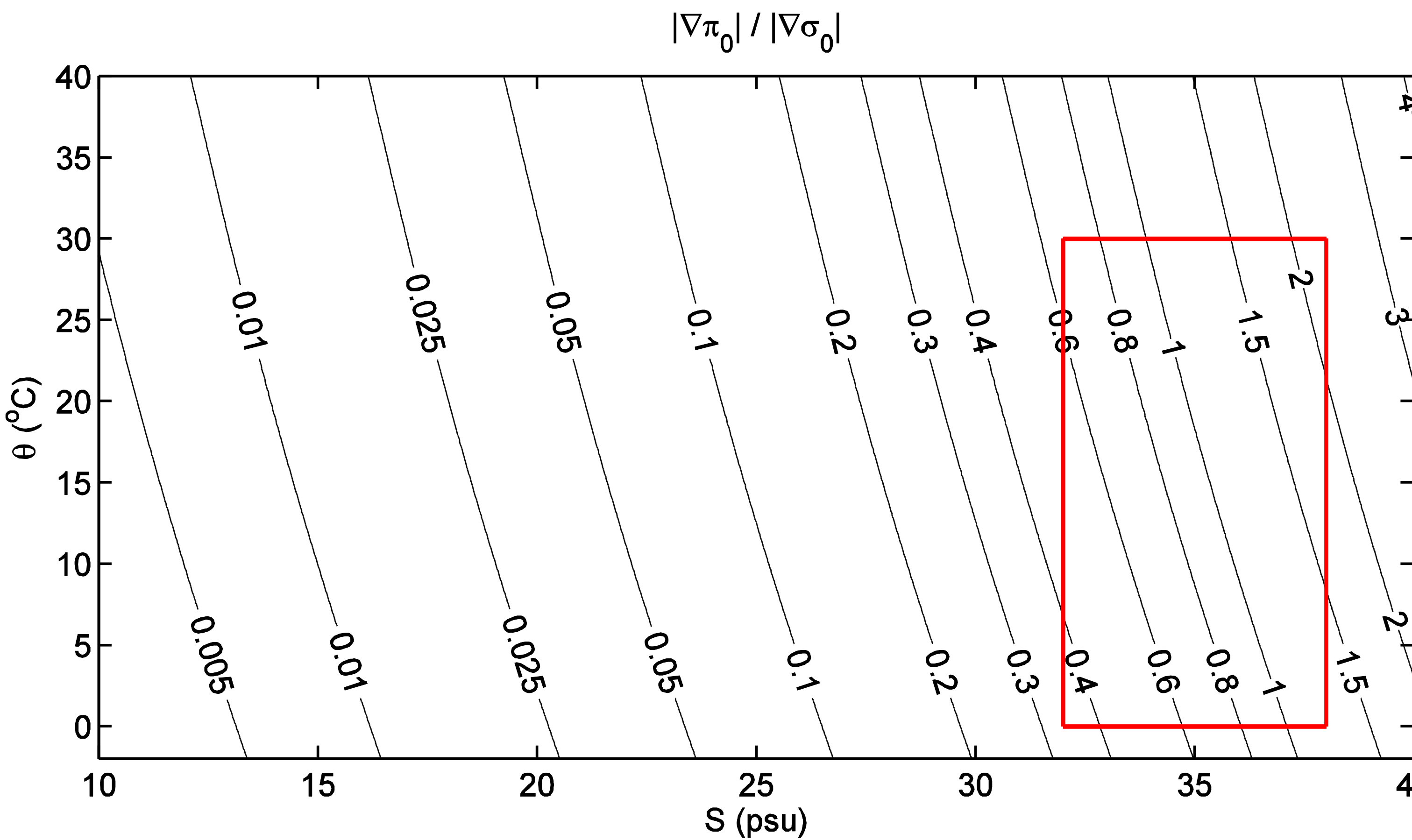



a) σ_0 & π_0 contours
θ (°C)
S (psu)
RMS=0.00090°
b) Deviation from orthogonality, in 0.001°
θ (°C)
S (psu)

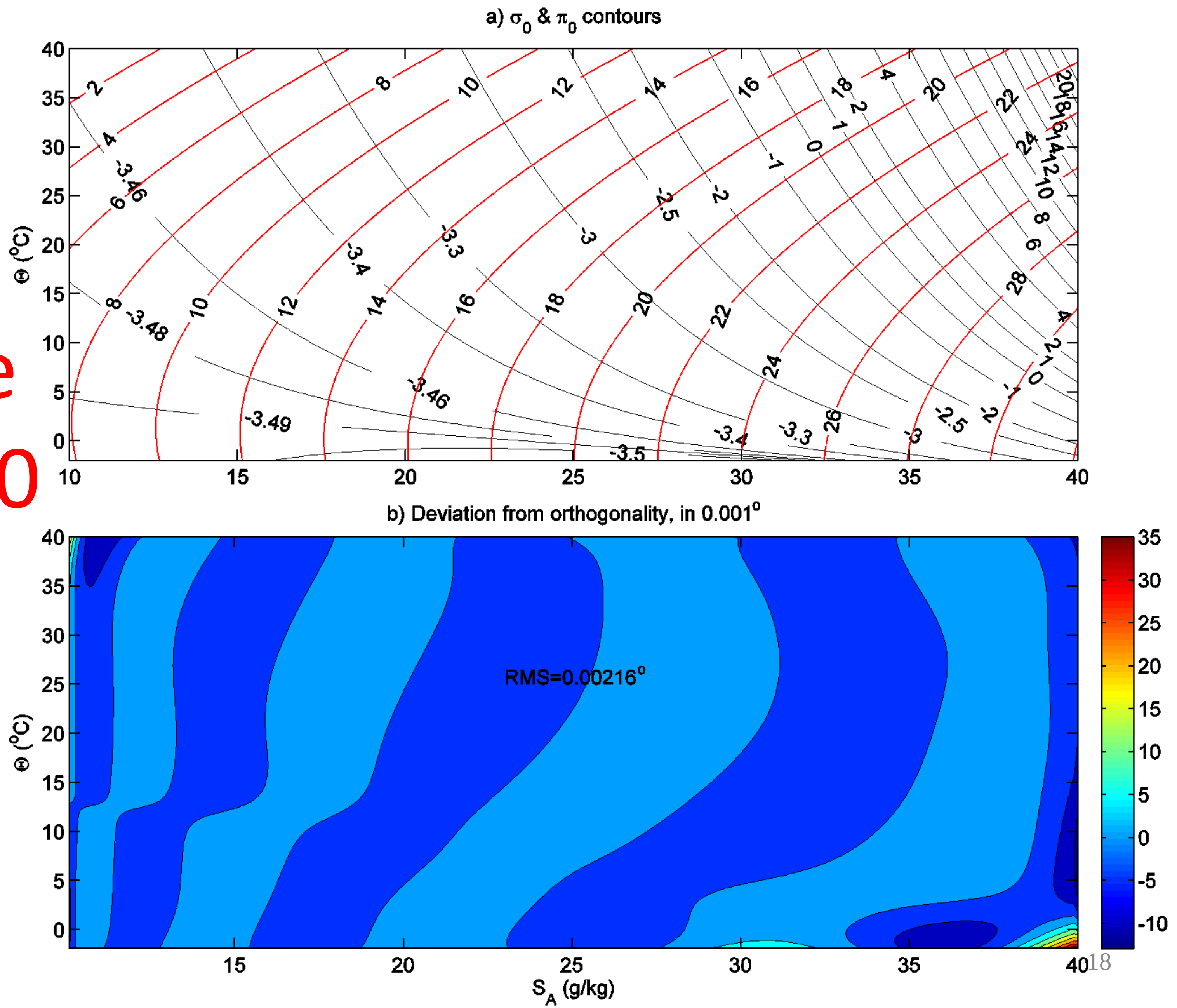
The amplitude ratio

$$\nabla\sigma = \rho_0(-\alpha\nabla\theta + \beta\nabla S)$$
$$\nabla\pi = \rho_0 c(\theta, S)(\beta\nabla\theta + \alpha\nabla S)$$

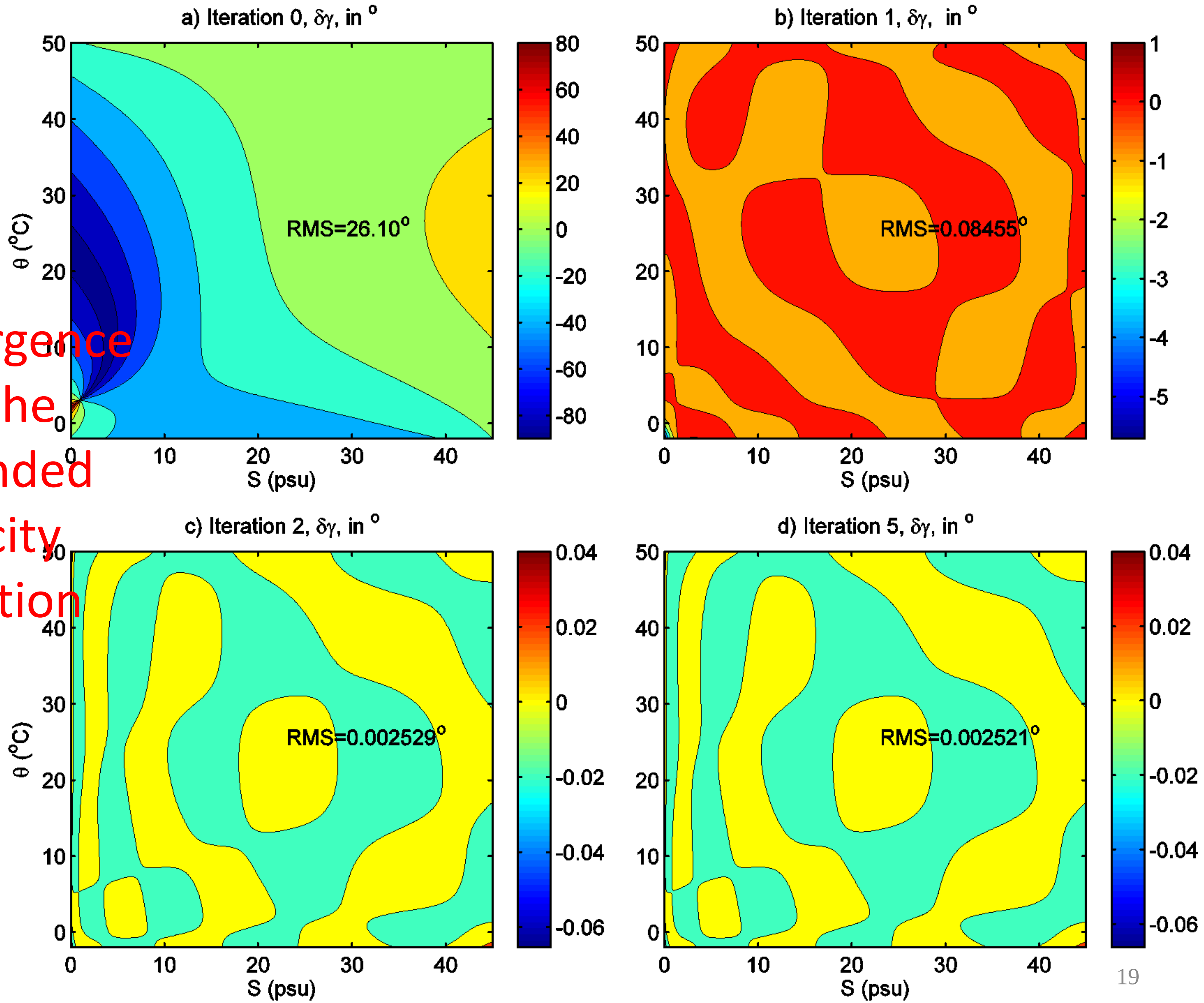
$c(\theta, S)$ cannot be defined apriori;
it is found from least-square method



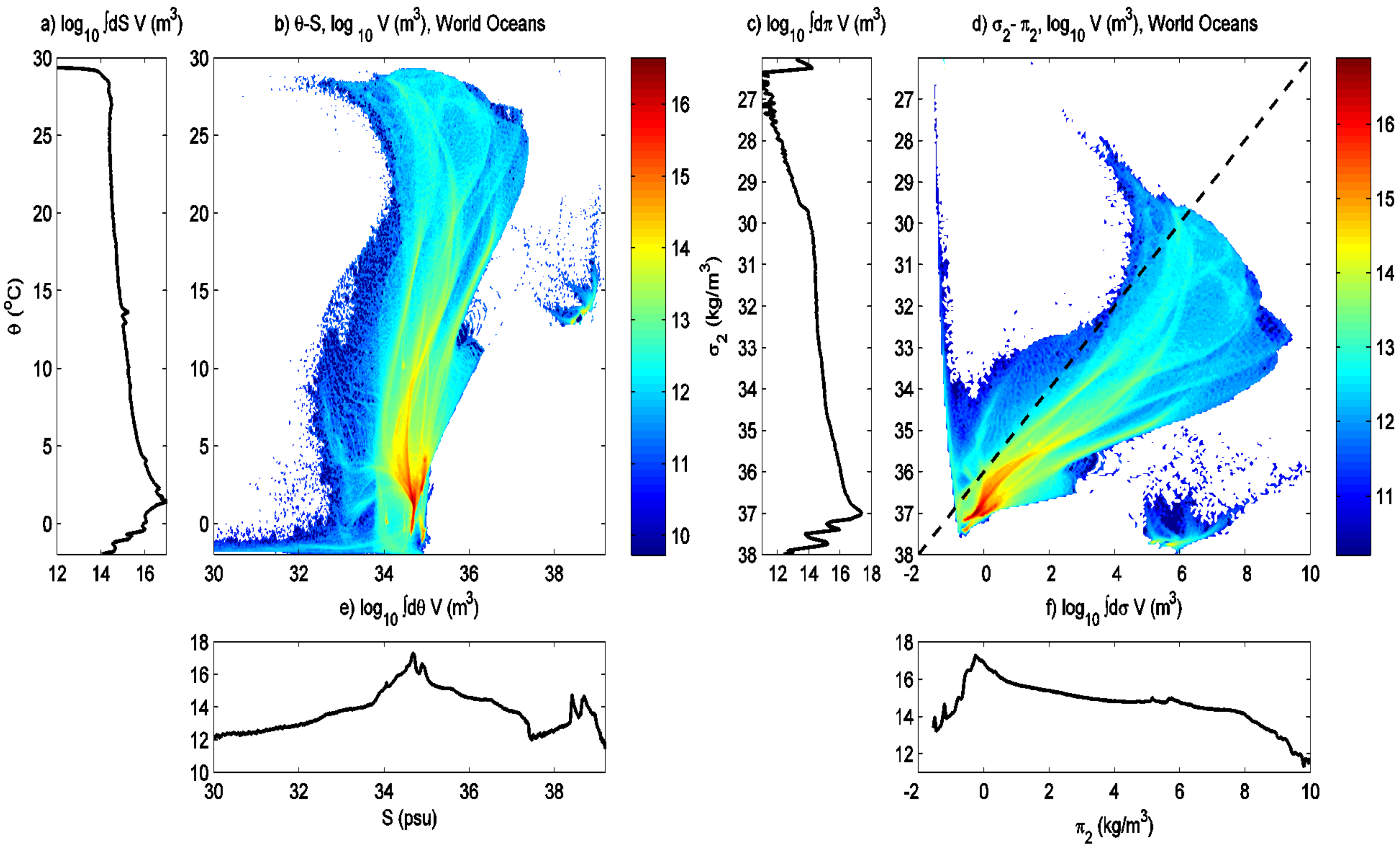
For the TEOS-10



Convergence of the extended spicity function



The volumetric distribution in T-S & σ−π diagram



The new challenge

- Traditional water mass analysis is based on T-S diagram: distance between water masses cannot be exactly defined…
- In the new $\sigma-\pi$ diagram, distance can be exactly defined → a more accurately way to define water mass properties in the world oceans.
- Due to global climate change, water mass properties in the world oceans may gradually shift → a more accurately way of classification is needed.
- Water mass analysis and classification based on the $\sigma-\pi$ diagram may be a new direction to go.

The meaning of spicity

- Warm and salty water is with high spicity

  又热又咸的海水是涩的:温度,盐度,密度,涩度

- All dynamical information of large-scale motion, in the horizontal and vertical directions, is contained in density

- Spicity is supplemental to density ---

  it describe information related to other aspects of motions, such as turbulence, thermohaline circulation…To be explored….

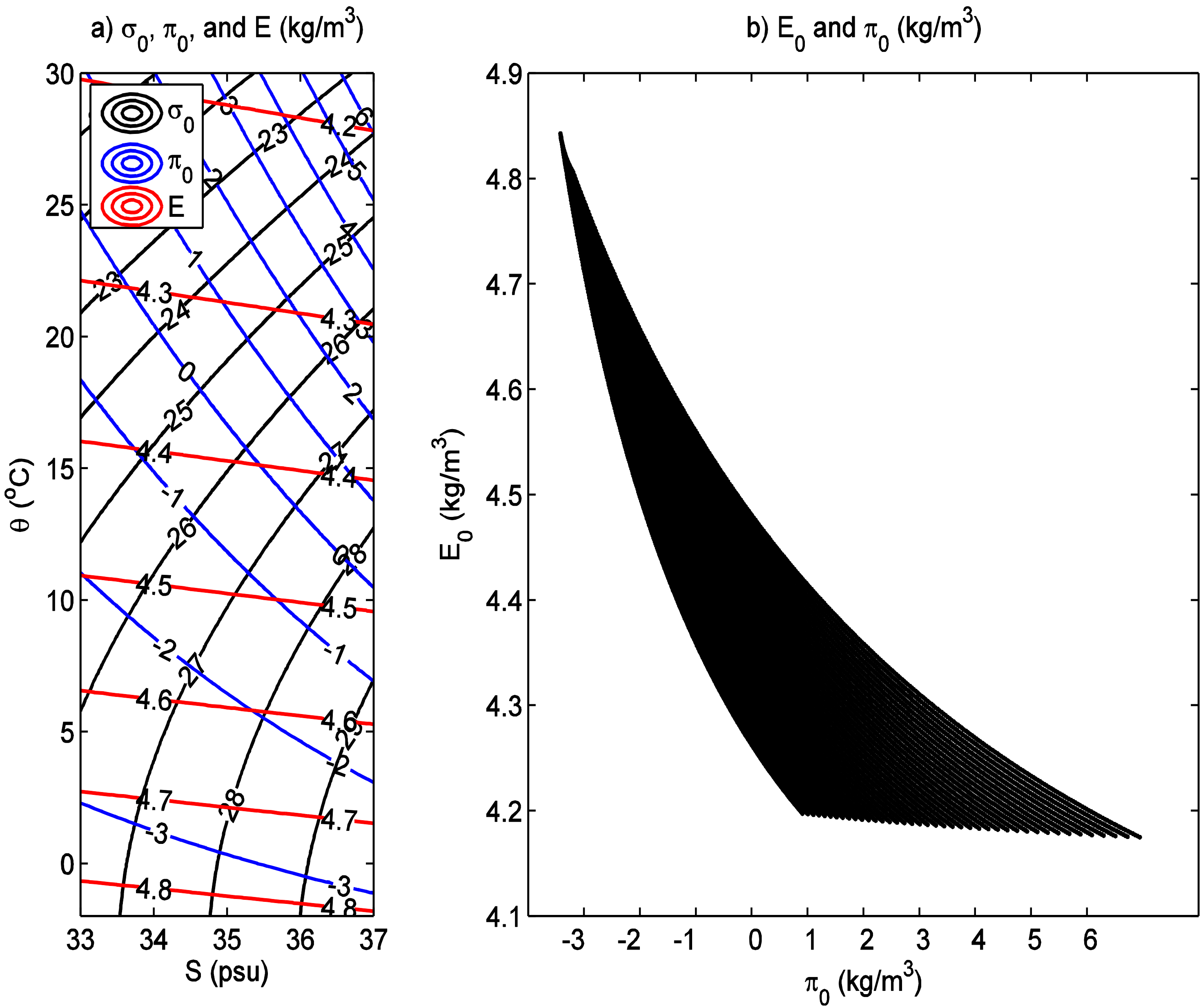
- Spicity may be important to ecology model ?

  Although density is the same, but warm/salty water is completely different for fishes …

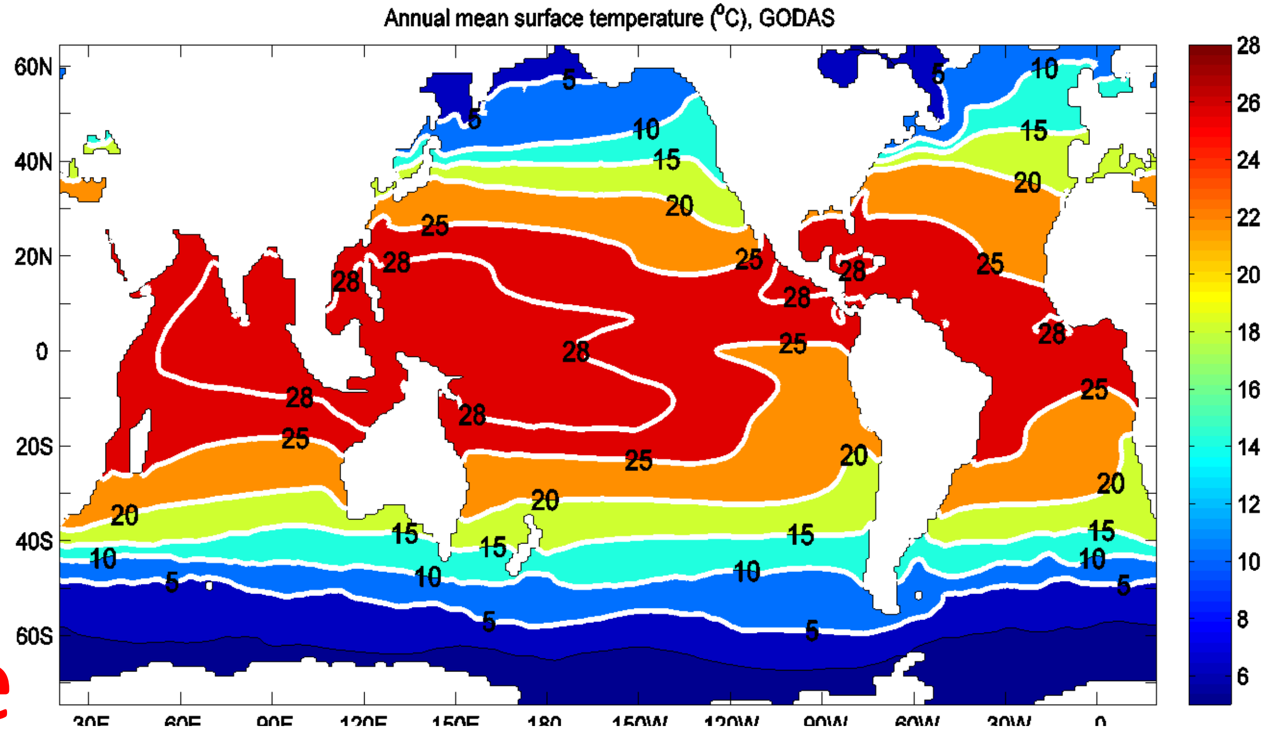
Spicity: a key parameter regulating thermohaline circulation and mixing

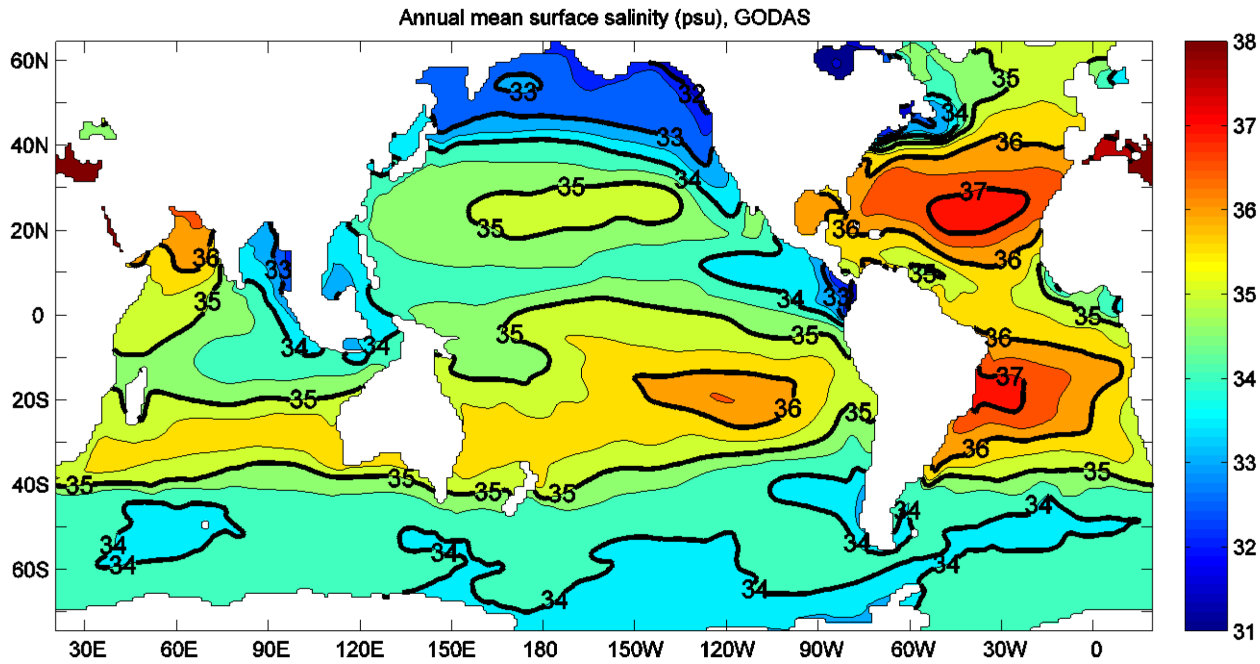
- If the density is a linear function of T&S → density alone determines the sinking overturning of the circulation
- Sea water density is a nonlinear function of T&S; thus, density alone does not control the sinking of deep water and thermohaline overturning
- Spicity regulates mixing, double diffusion, thermohaline intrusion and turbulence

Spicity is a good measure for compressibility



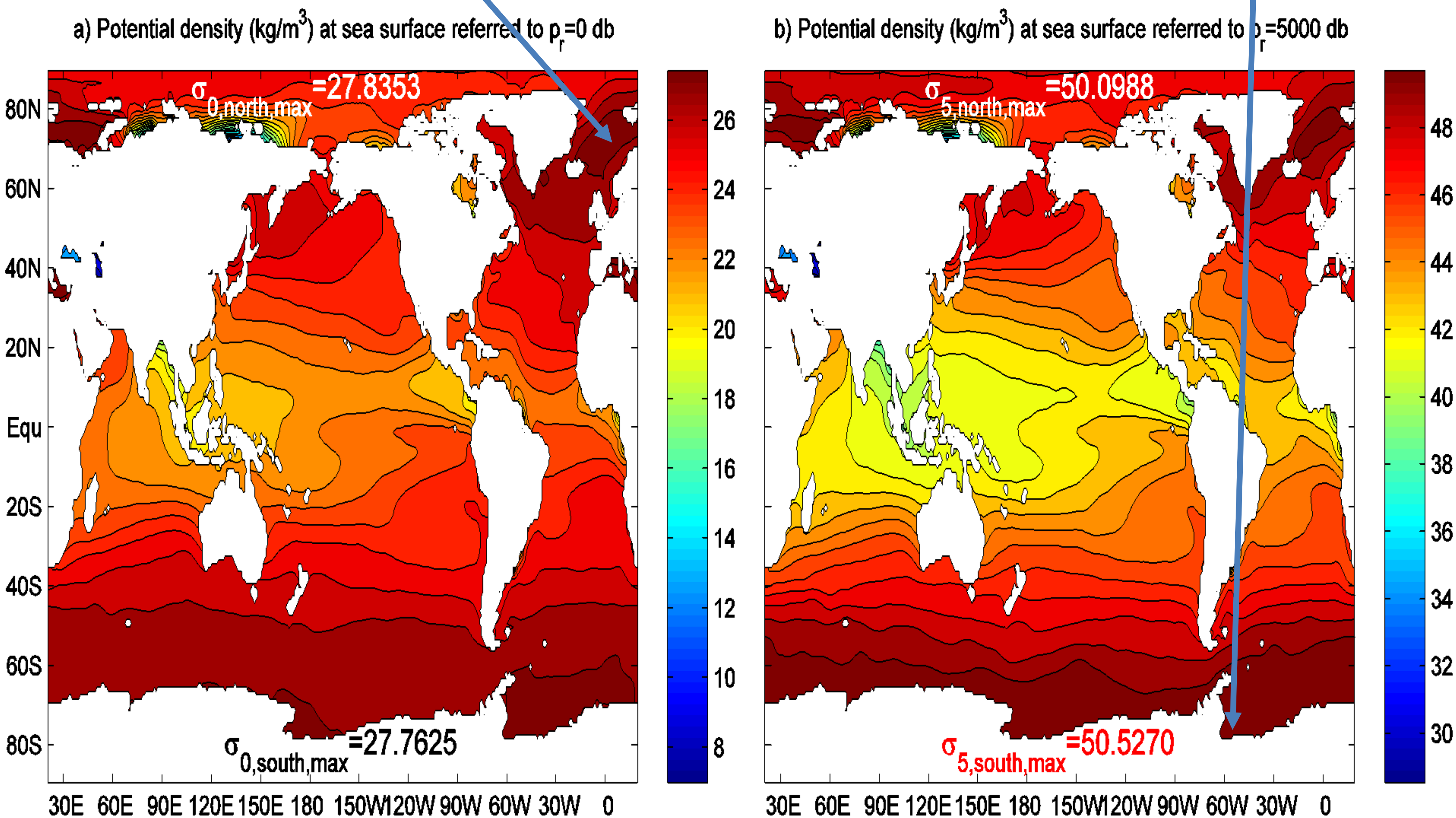
Annual mean temperature & salinity



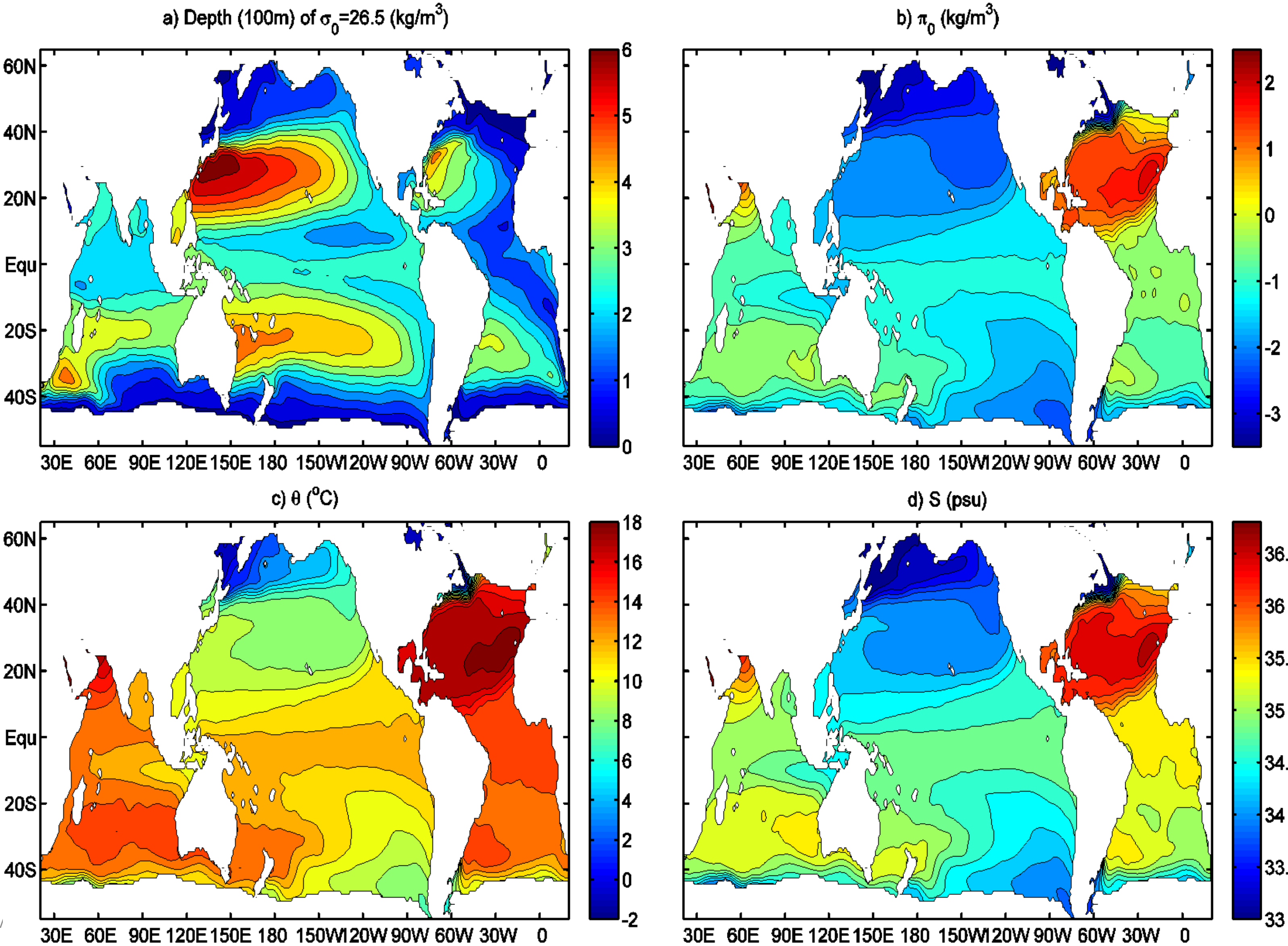


On the sea surface, the heaviest water is in N. Atl

Using 5000db as reference pressure, the heaviest surface water is in Wedell Sea (S. Atl)



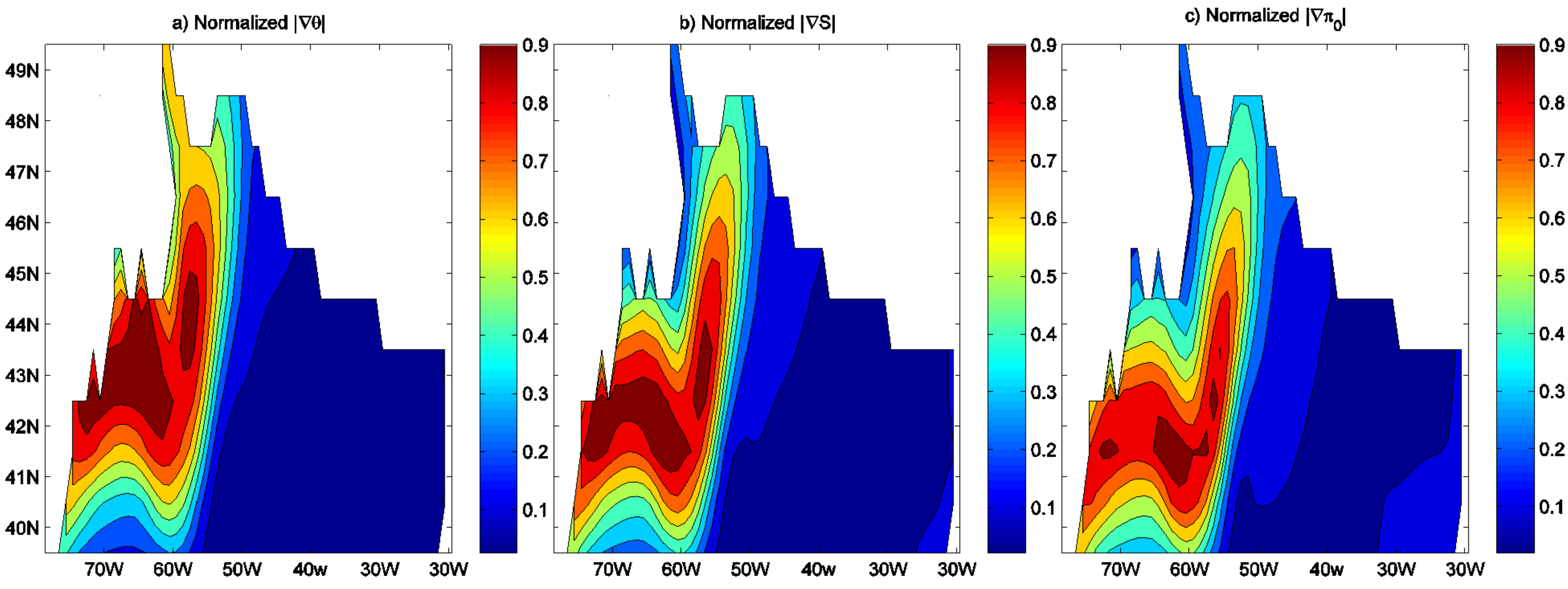
T & S are density compensated on isopycnal surfaces



π is a nonlinear function of T,S→ their variabilities in space/time are different

- In many papers, people collect observations and project the data onto isopycnal surface; then use [T, S, or η – spice ($\eta = \rho_0(\alpha T + \beta S)$)] to analyze climate variability or power spectrum of internal waves, submesoscale currents and turbulence
- In theory, the most accurate way to study variability on isopycnal surface should be based on spicity --- any other thermodynamic parameter can lead to contaminated results

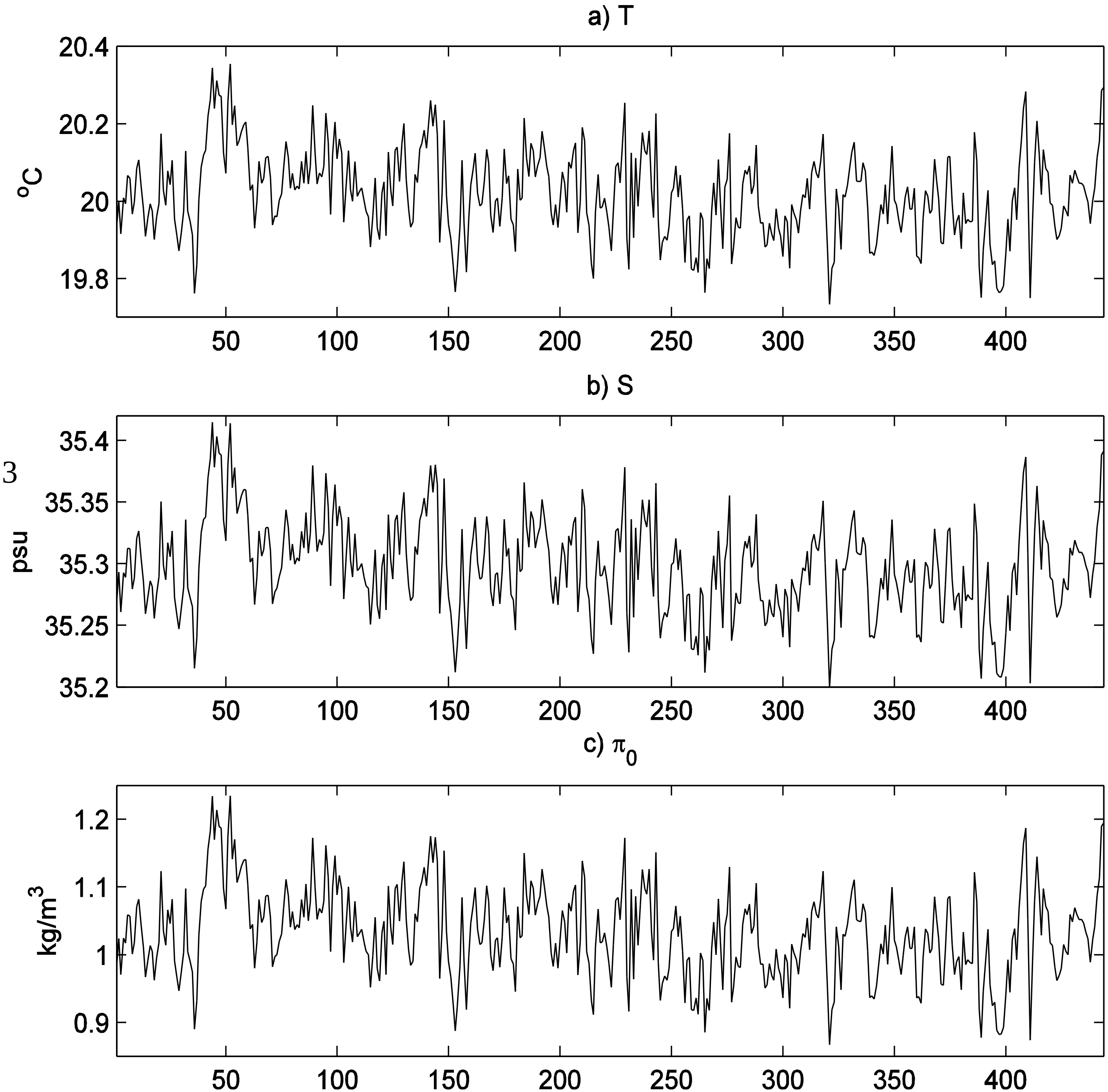
T,S on isopycnal surface are density compensated→ the distribution of (T,S,π) look similar; but, they are really different



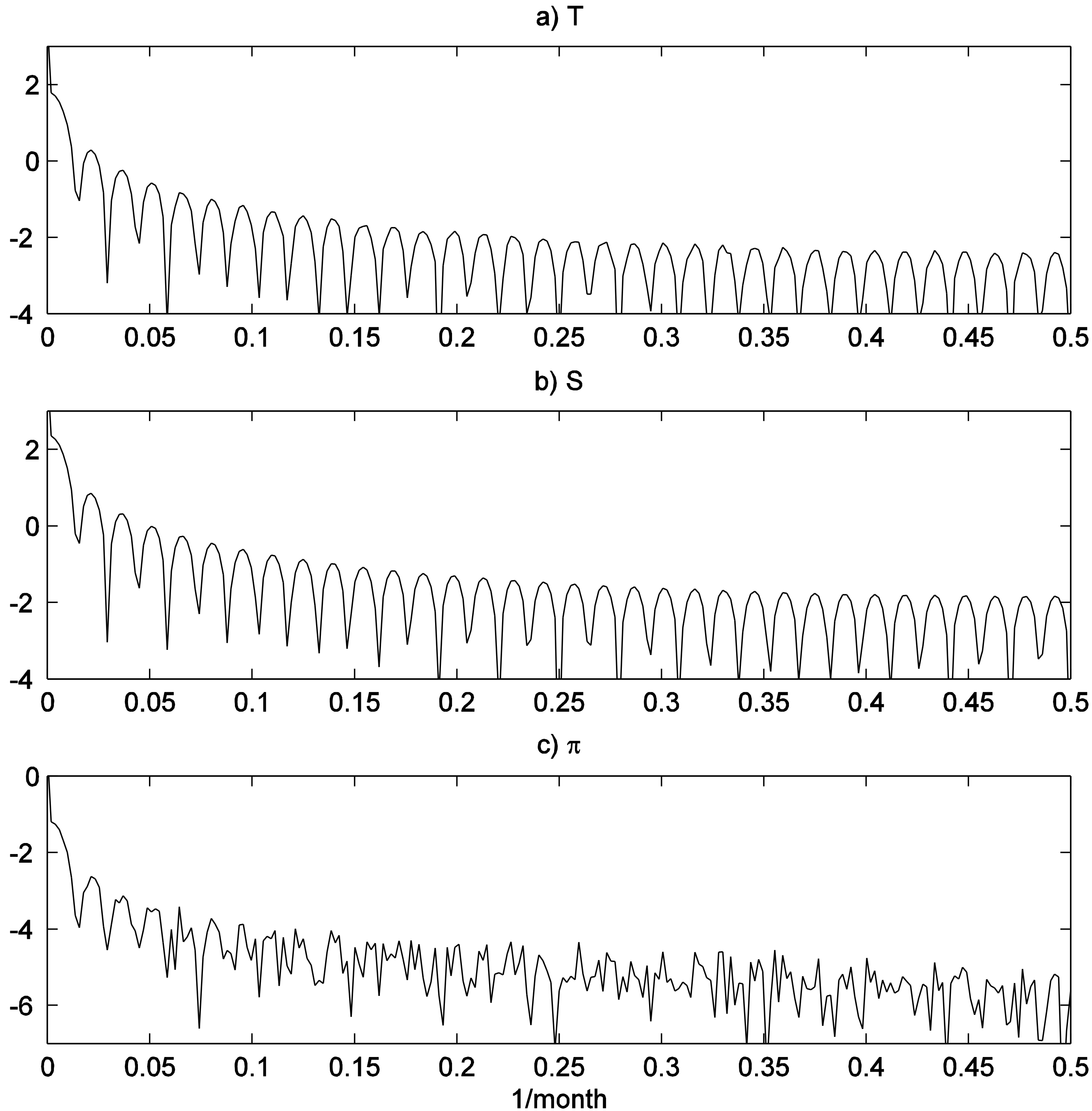
Signals on isopycnal surface

$\sigma_0 = 25.0kg/m^3$

They look similar but not exactly



Power spectrum based on monthly data on isopycnal are different



Density-spicity plane is a metric space

In the classical Euclid geometry

we can define 0th , 1st, 2nd moment, area, distance, radius of curvature.

"Distance" between water masses?

- The traditional θ-S diagram? No!!
  1) θ and S has different units (dimensions)
  2) α and β vary with θ and S.
- (σ,π) have the same dimension → distance can be defined in $\sigma-\pi$ diagram.

Potential spicity and Radius of signal

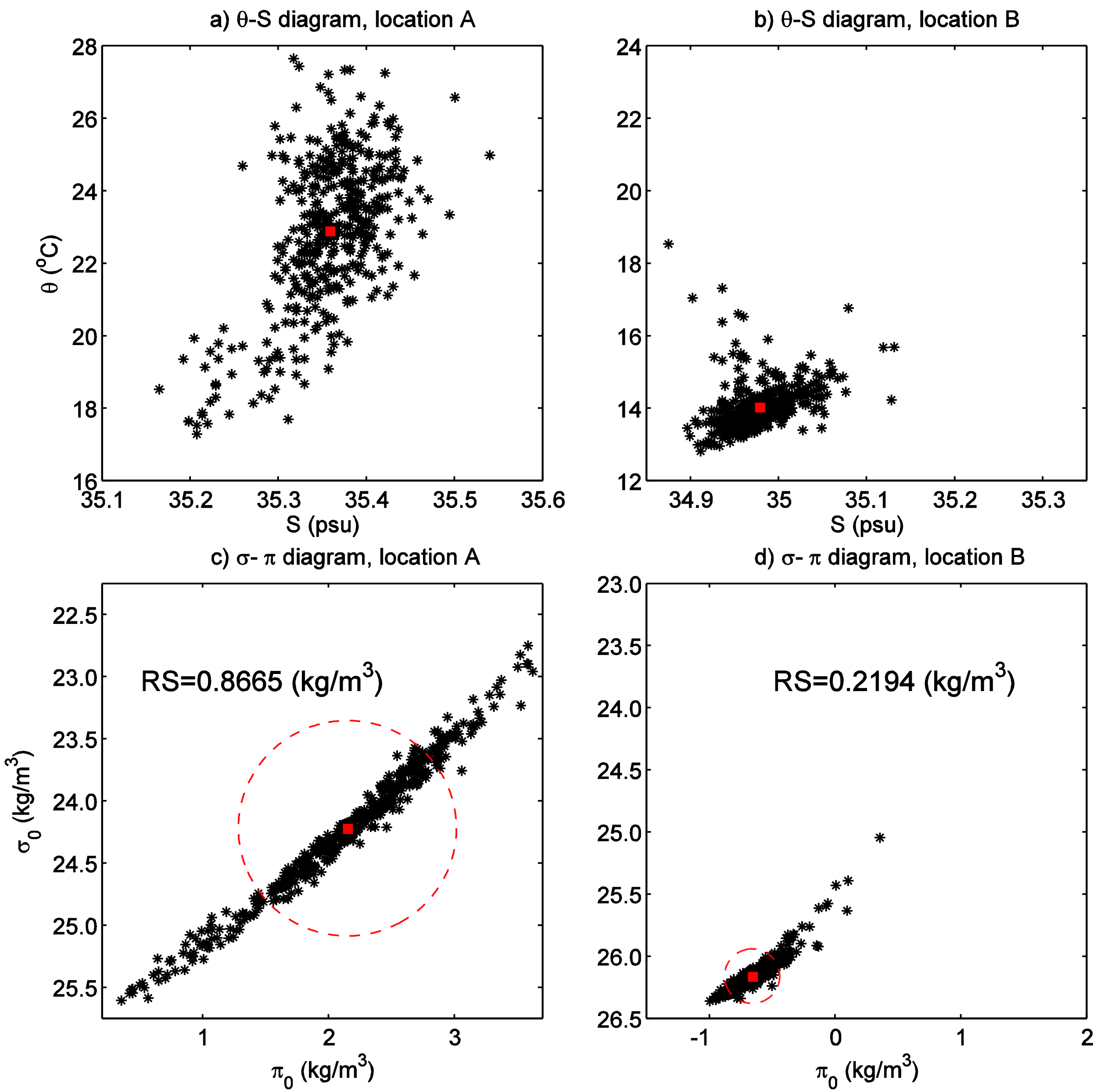
- Potential density and potential spicity form an orthogonal coordinates.
- The distance is defined : $R = \sqrt{(\sigma_1 - \sigma_2)^2 + (\pi_1 - \pi_2)^2}$
- Radius of signal: for a set of signal

$$Rs = RMS\left[\sqrt{(\sigma - \bar{\sigma})^2 + (\pi - \bar{\pi})^2}\right]$$

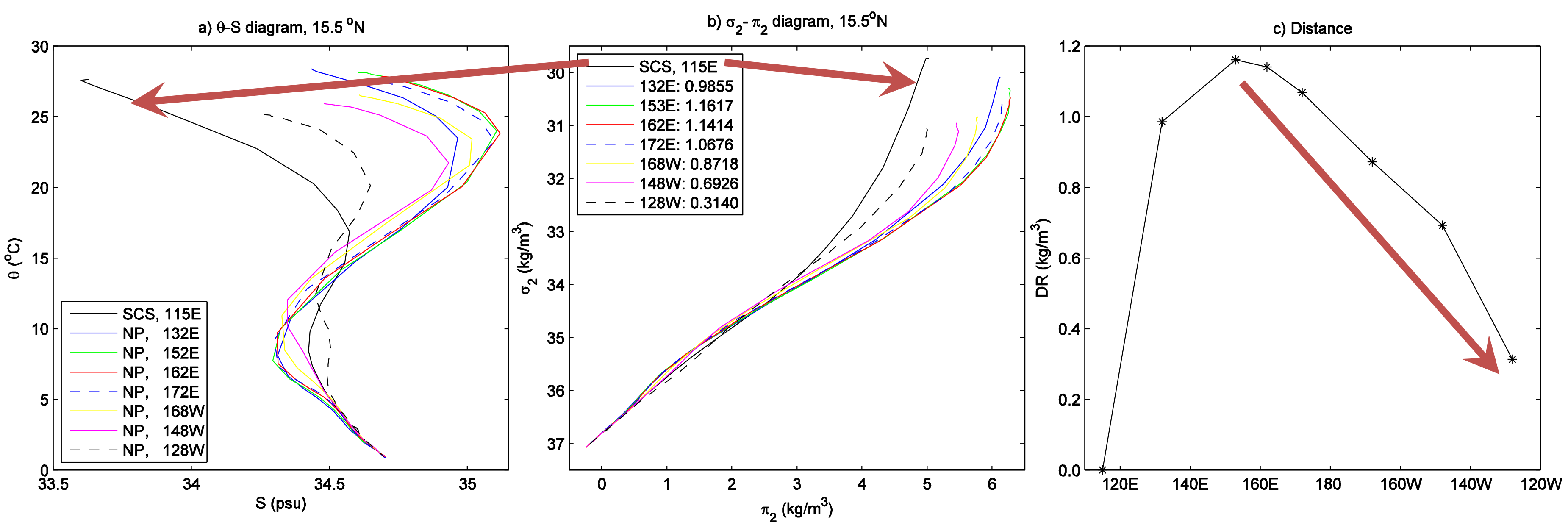
- Distance between two σ–π curves is defined as the RMS($\Delta\pi$) over the density range shared by both σ–π curves.
- Old approach: trace (T,S) on isopycnal
- New approach: minimum $\delta\pi$ on multi isopycnal surfaces, use least square distance as a measure

Radius of signals

Two stations along the equator (0.5ºS) taken from GODAS
Station A: 179.5ºE
Station B: 100.5ºW
At depth of 145m.

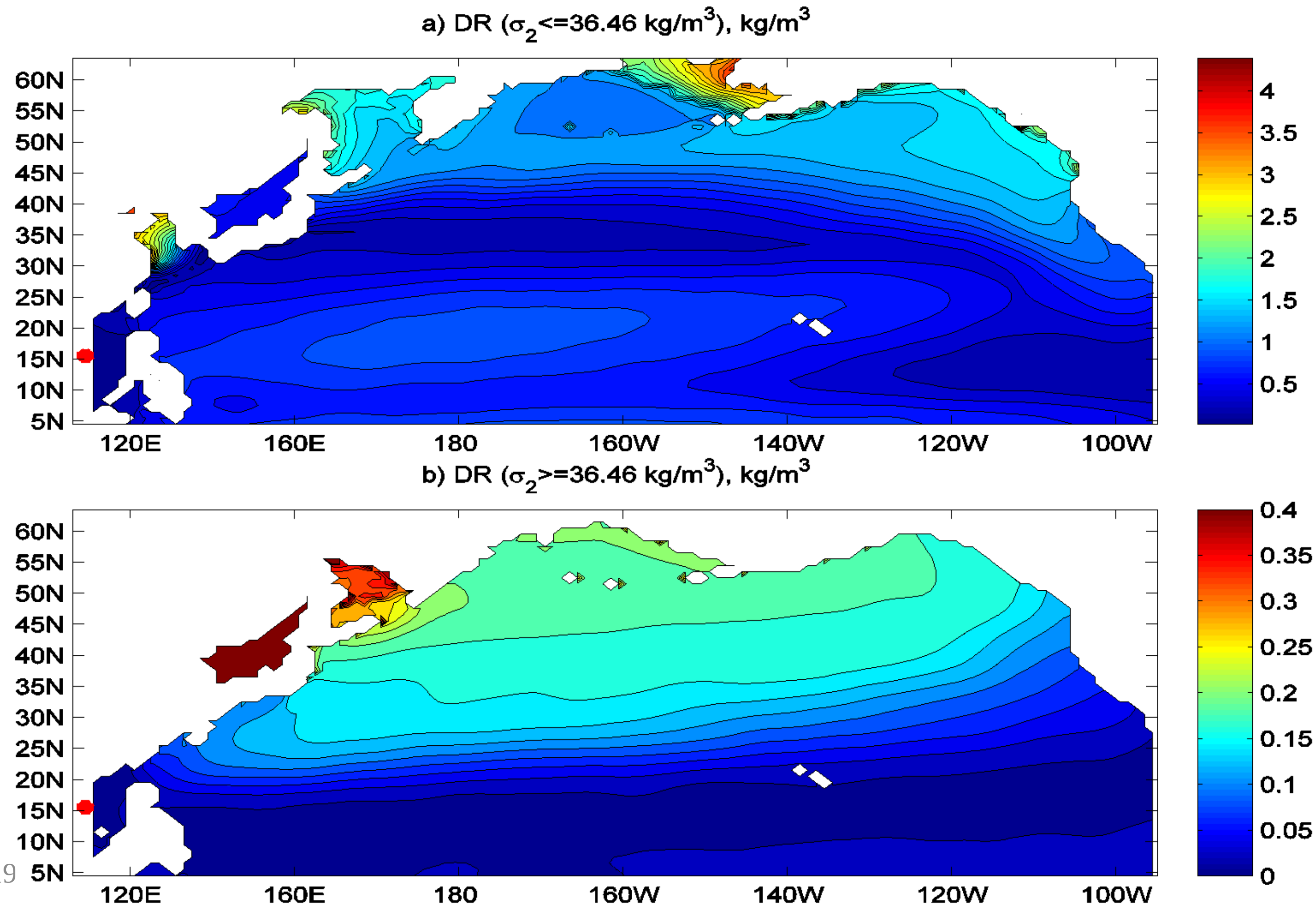


The distance between σ–π curves is defined as the RMS(Δπ) over the density range shared by both σ–π curves

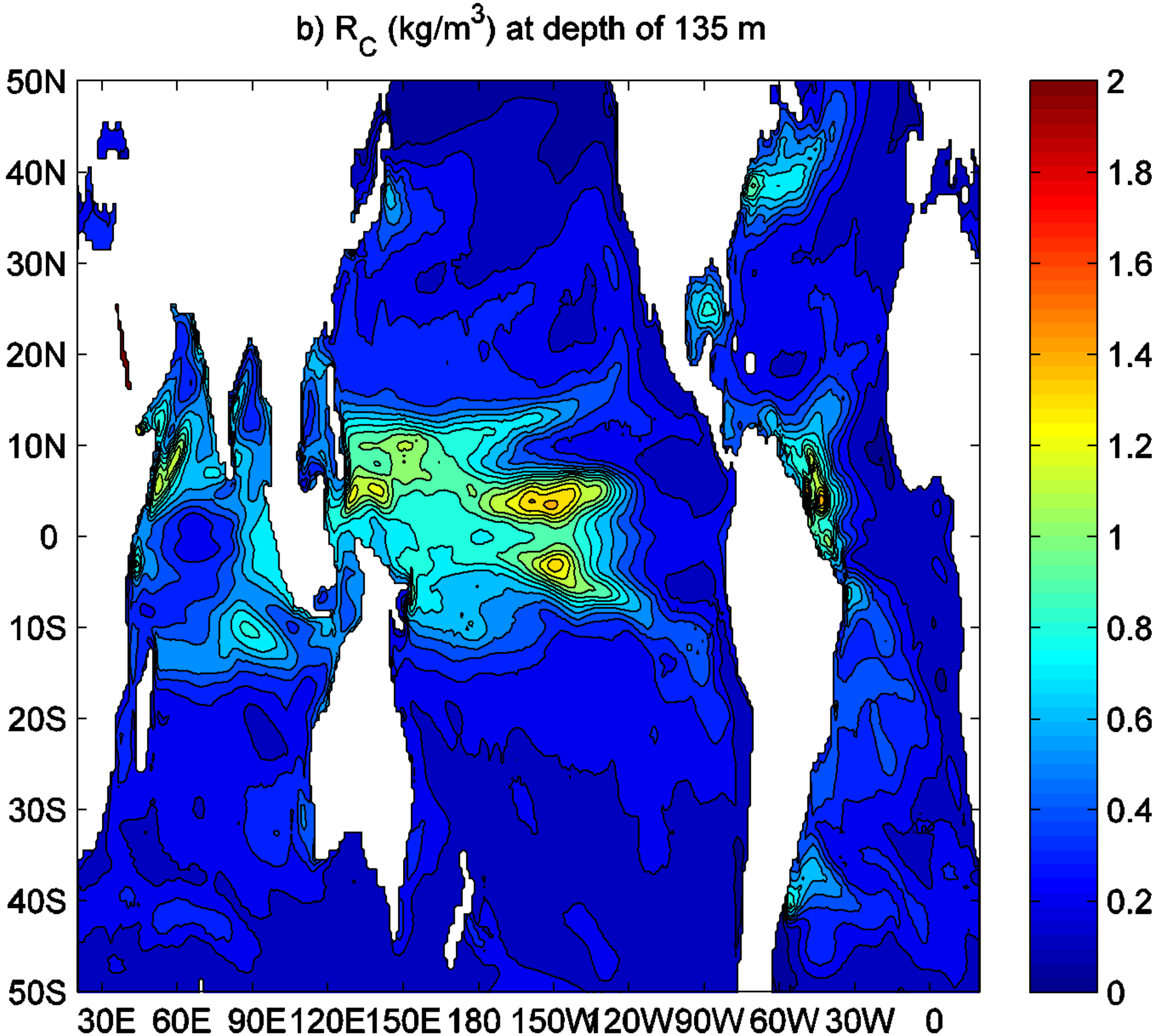


The distance between two σ–π in SCS and N. Pacific

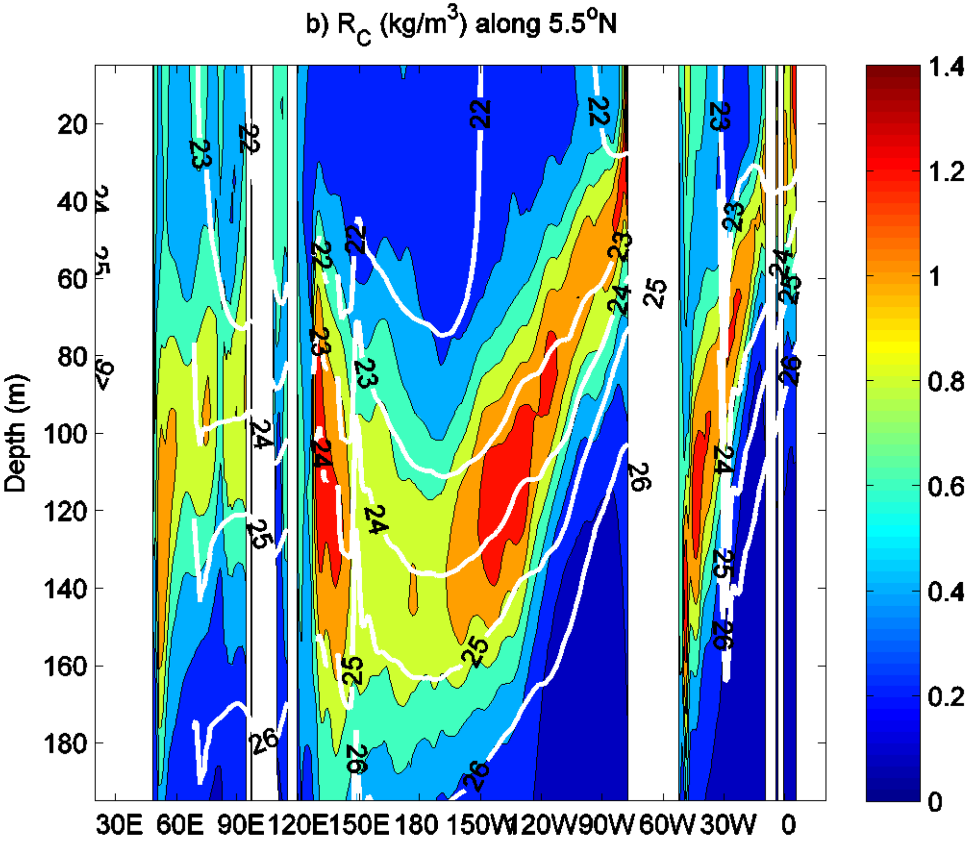
This method may be used to identify the source region of an eddy

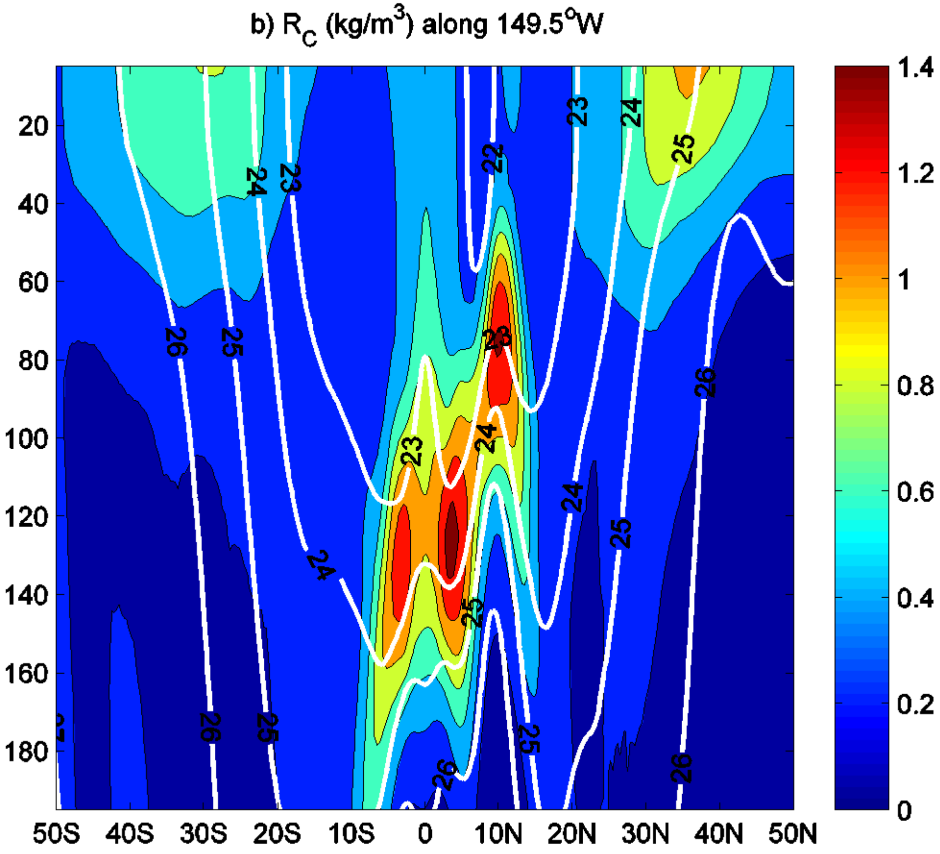


2) Radius of climate radius of state

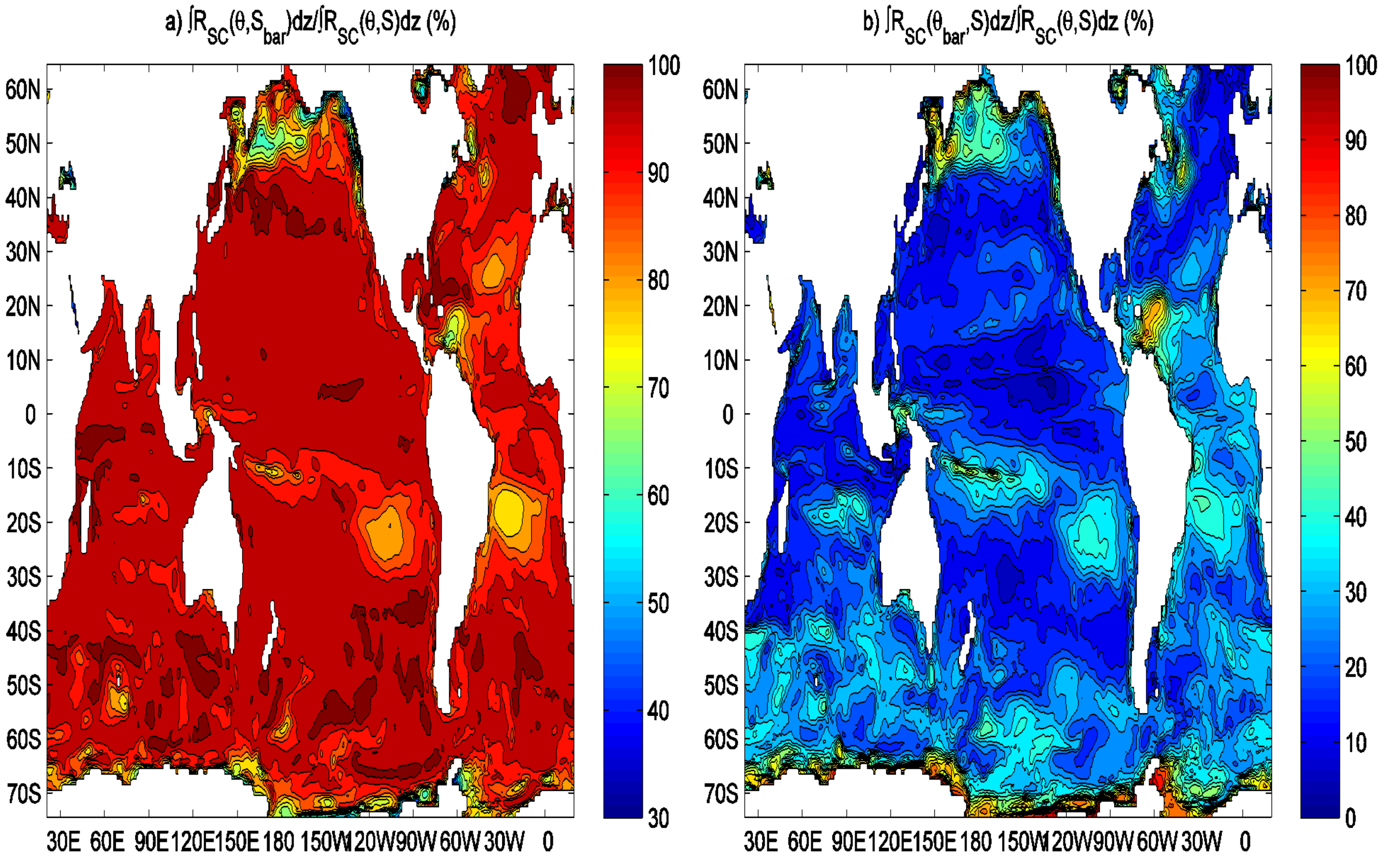


Zonal and meridional sections



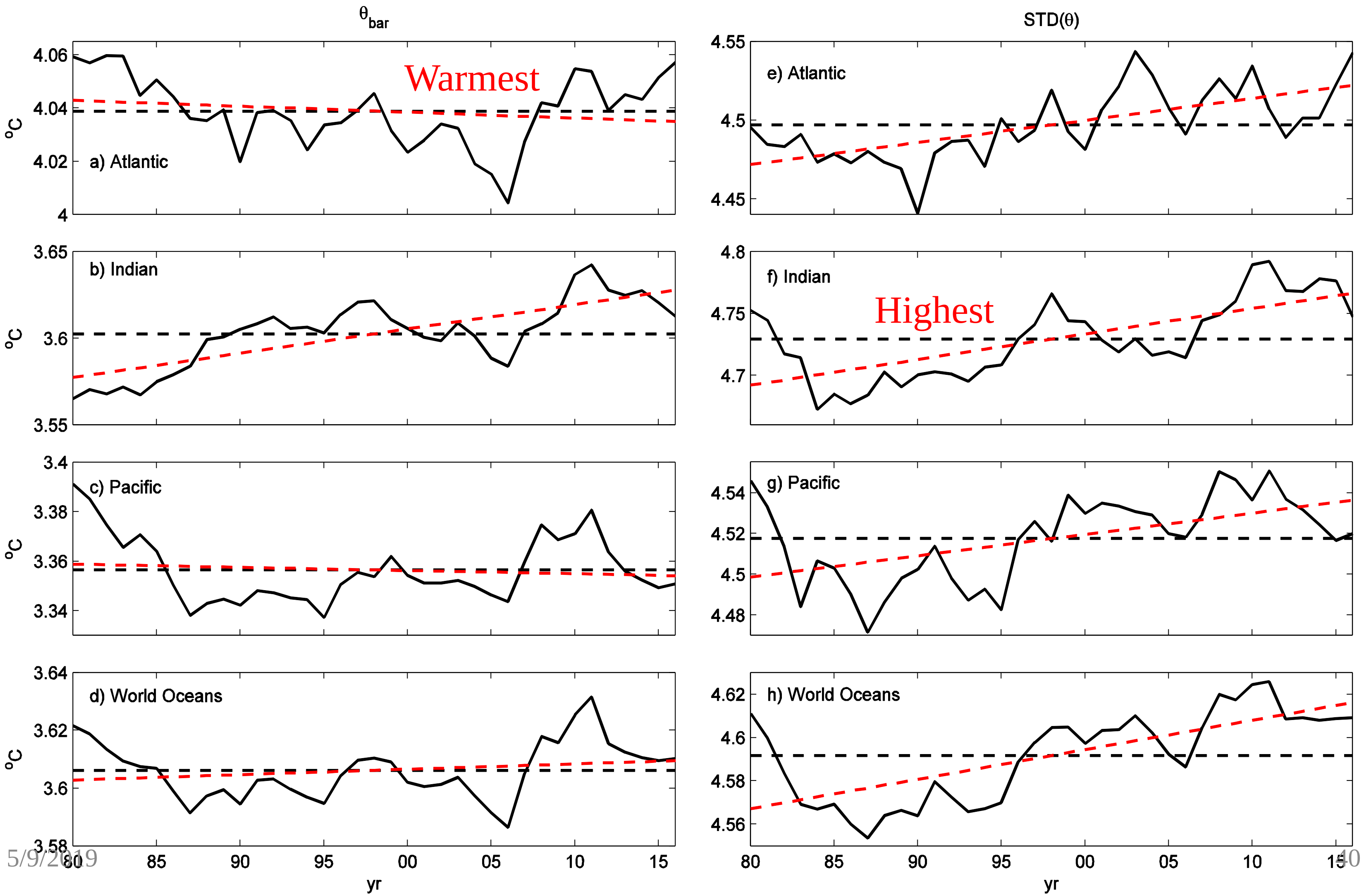


Contributions due to T & S



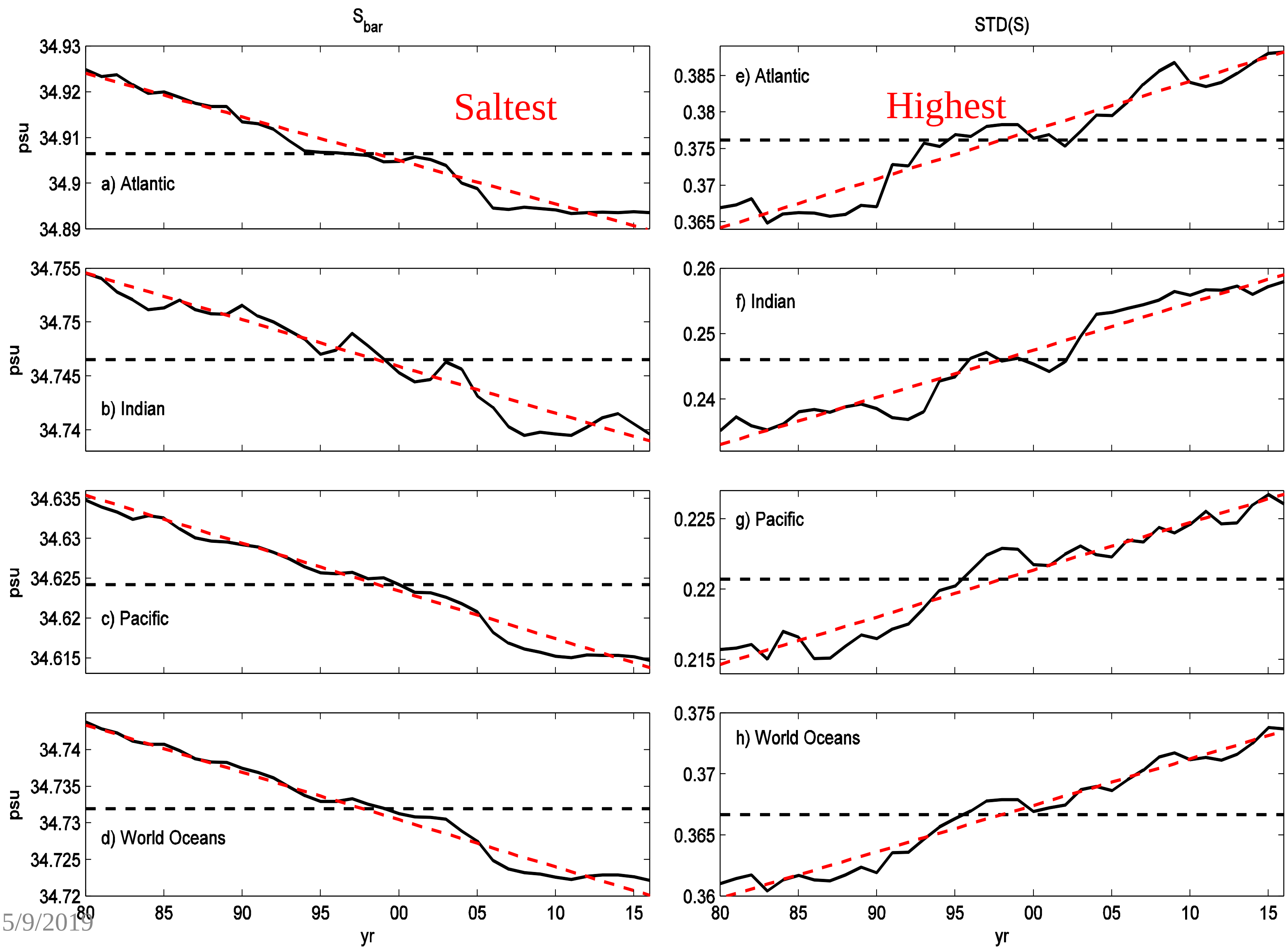
Evolution of poten. Temp (GODAS)

Global warming & increase in T variability, in particular in the Indian
Does this lead to more active circulation in Indian Ocean? No !



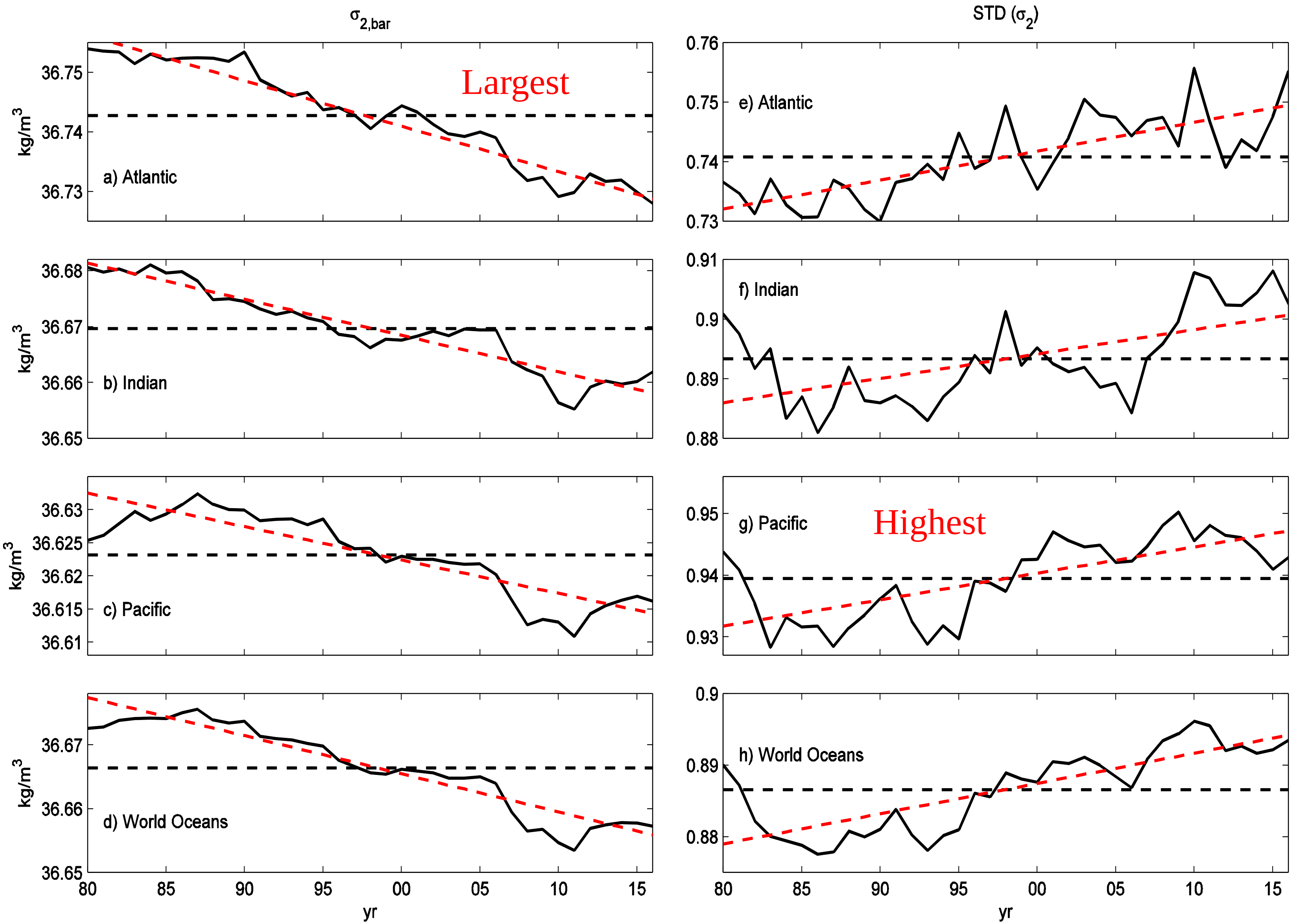
Evolution of Salinity (GODAS)

Melting of glaciers, freshening but increase in S variability

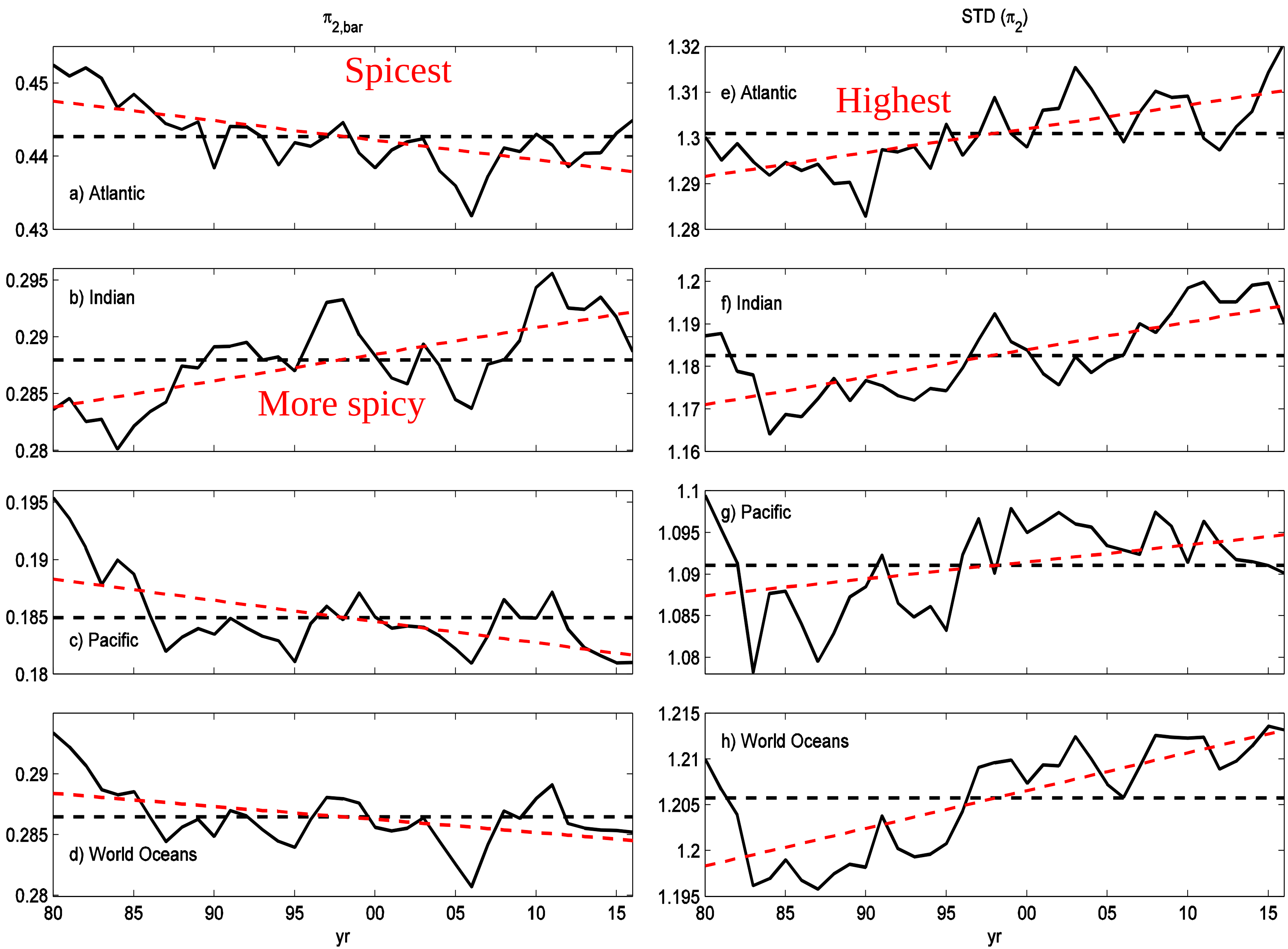


Evolution of $\sigma_{2,bar}$ (GODAS)

Becomes lighter, but variability is enhanced

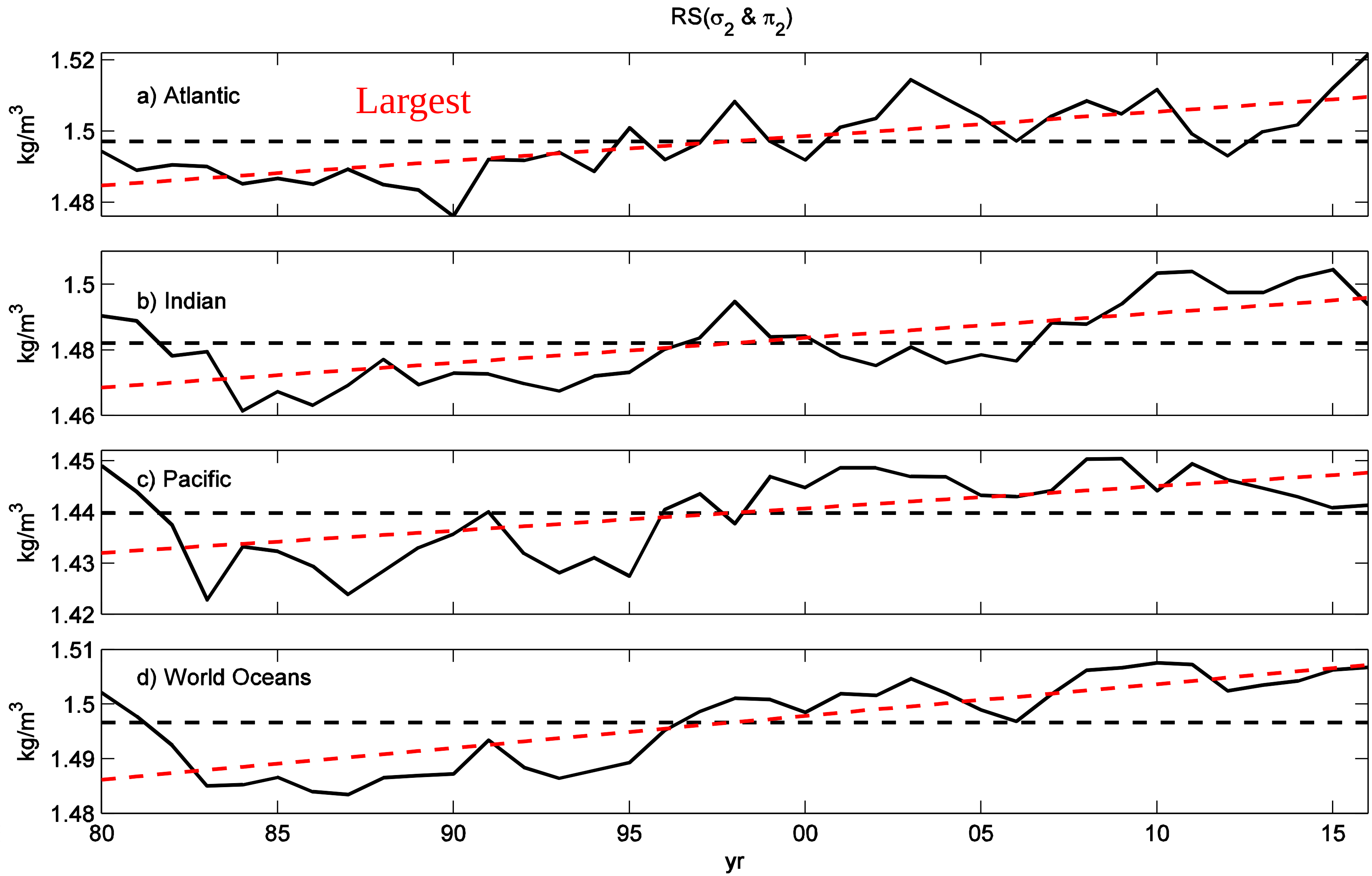


Evolution of $\pi_{2,bar}$ (GODAS)

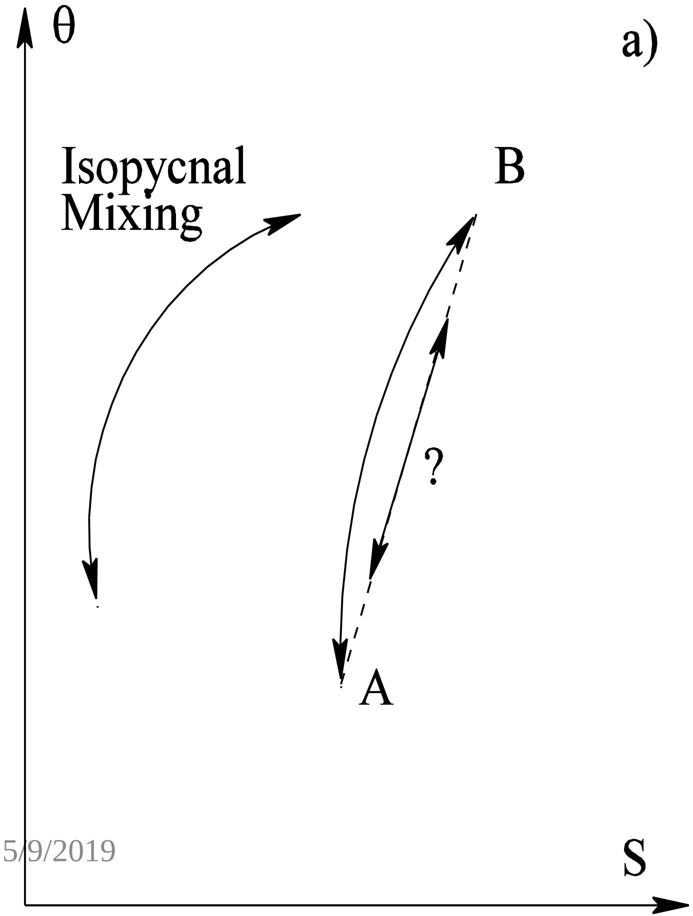


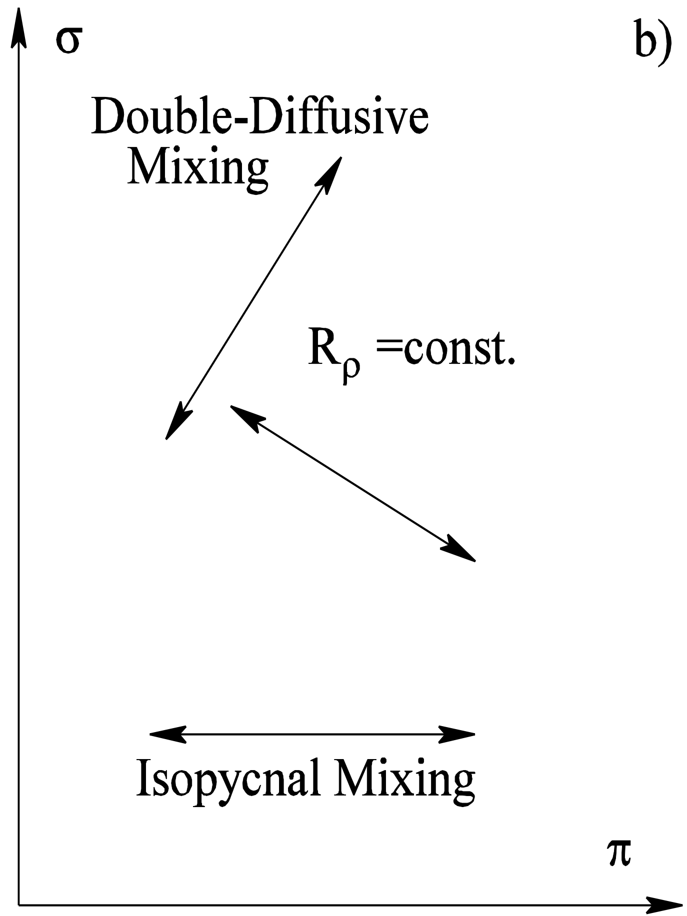
Evolution of the radius of state
Variability of THC increases !!

RS in Atlantic is the largest → the most active circulation

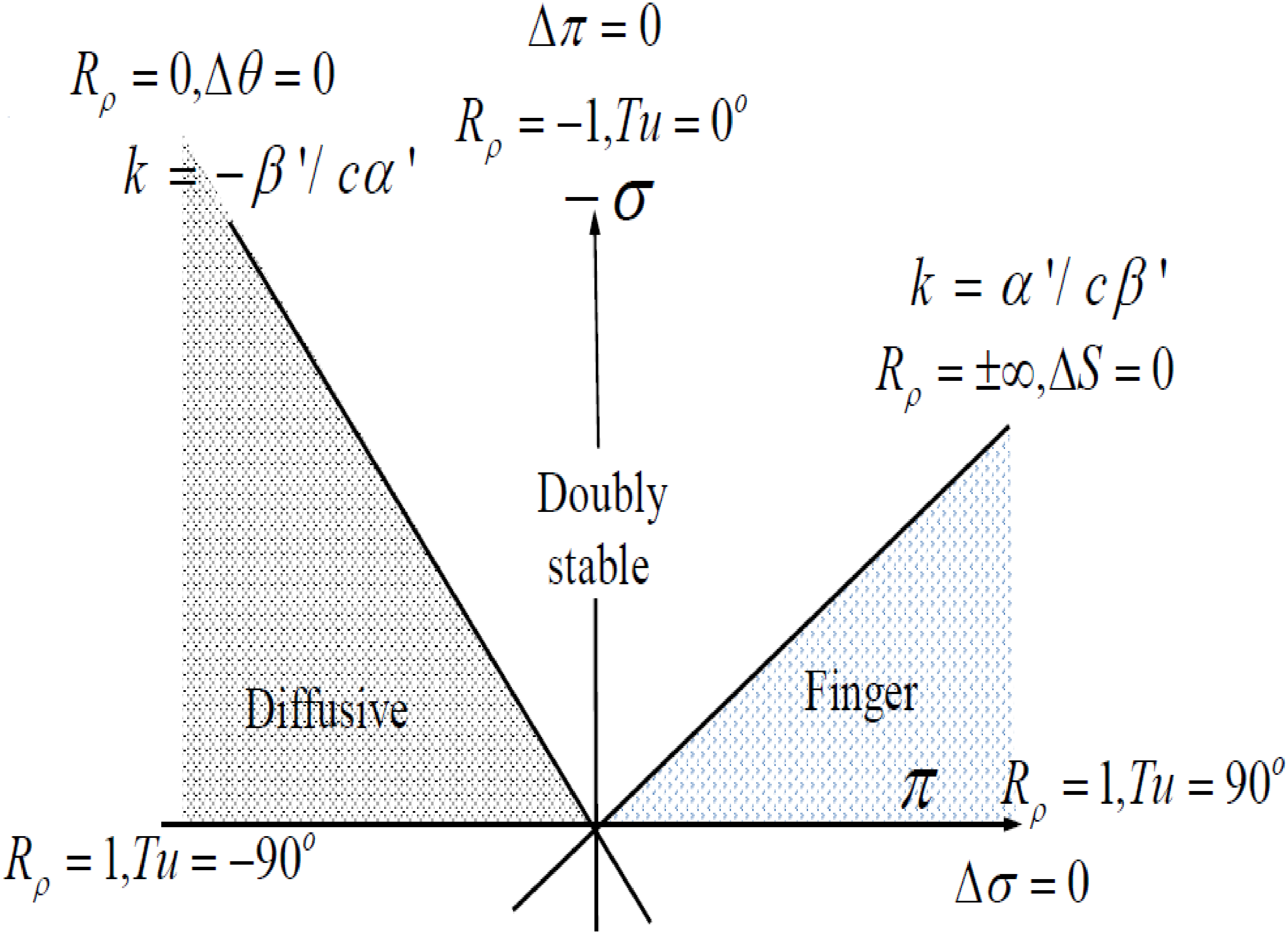


3) Density ratio
Mixing along a straight lines in T-S diagram does not happen in the oceans

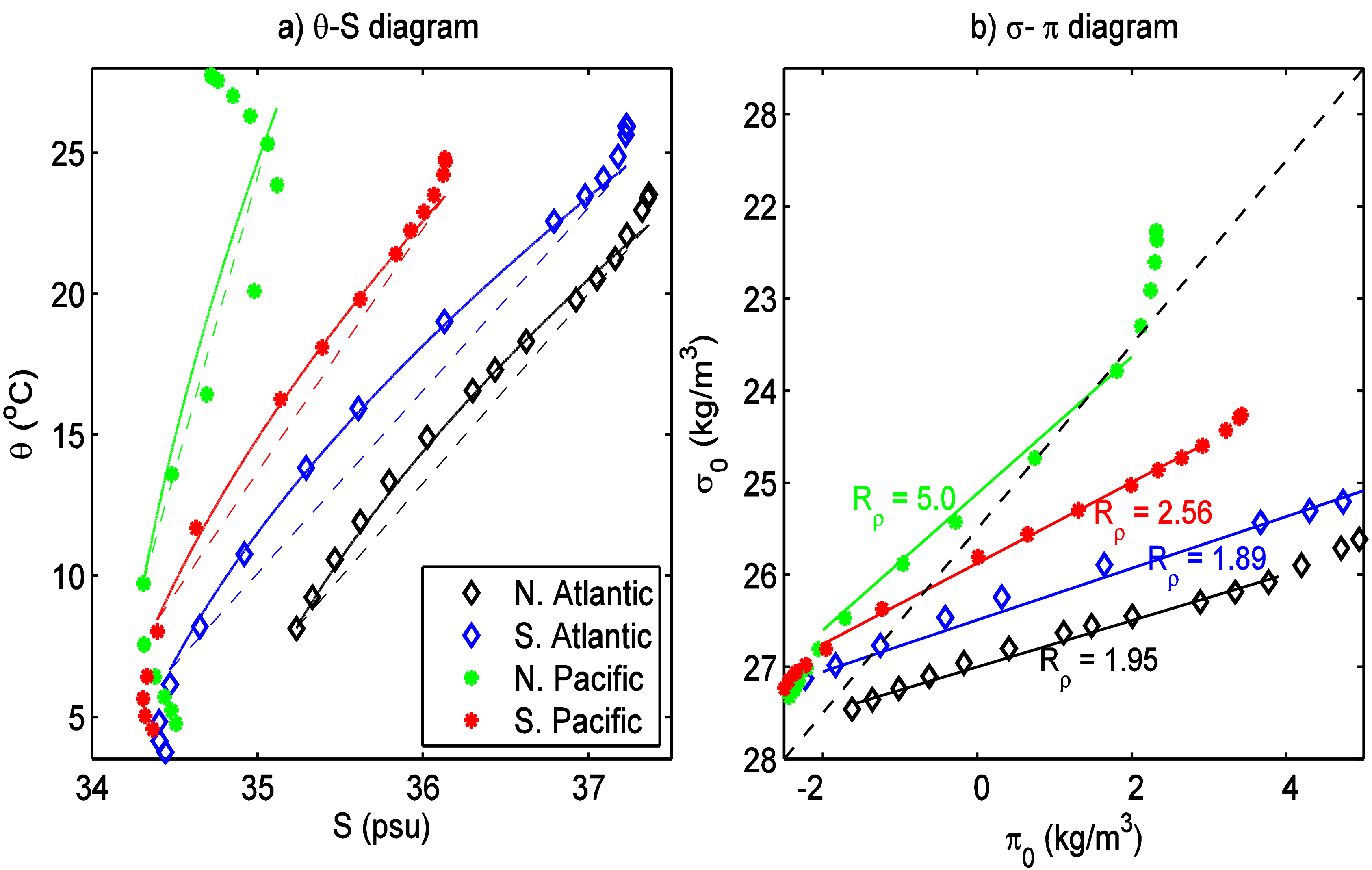




Different modes of double-diffusion mixing identified from the $\sigma - \pi$ diagram

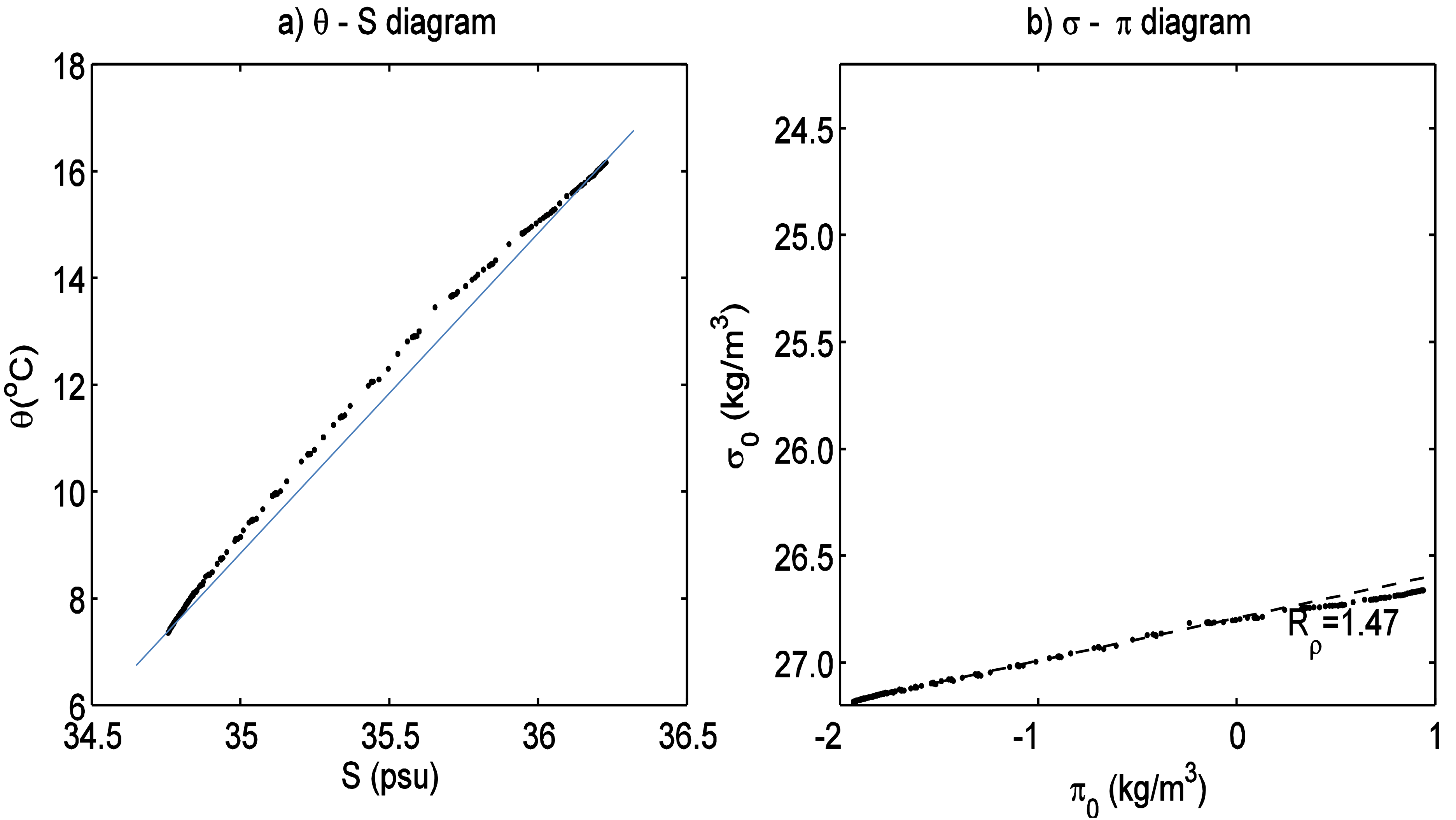


Central water in the world oceans appear as straight lines in σ–π diagram

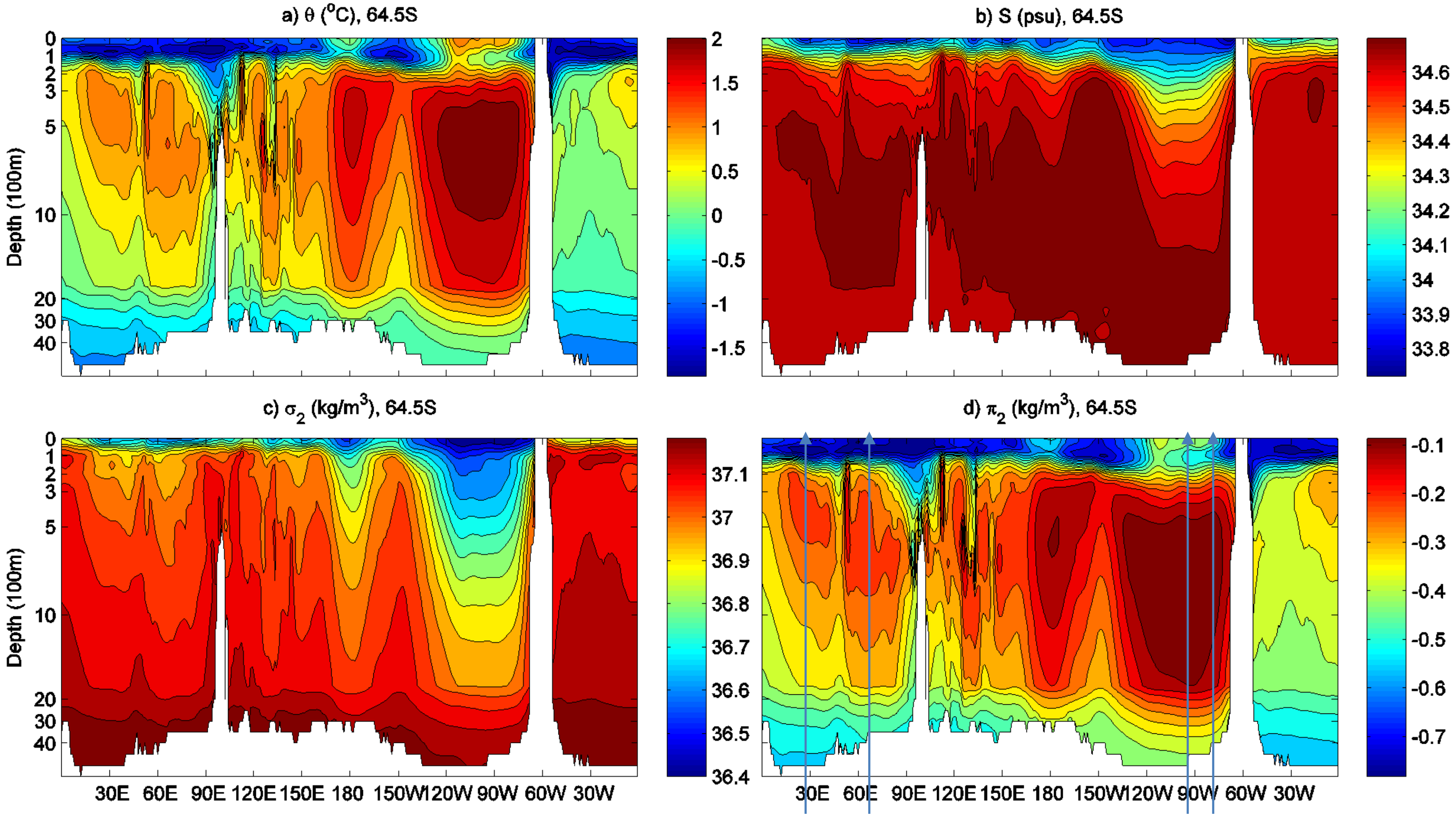


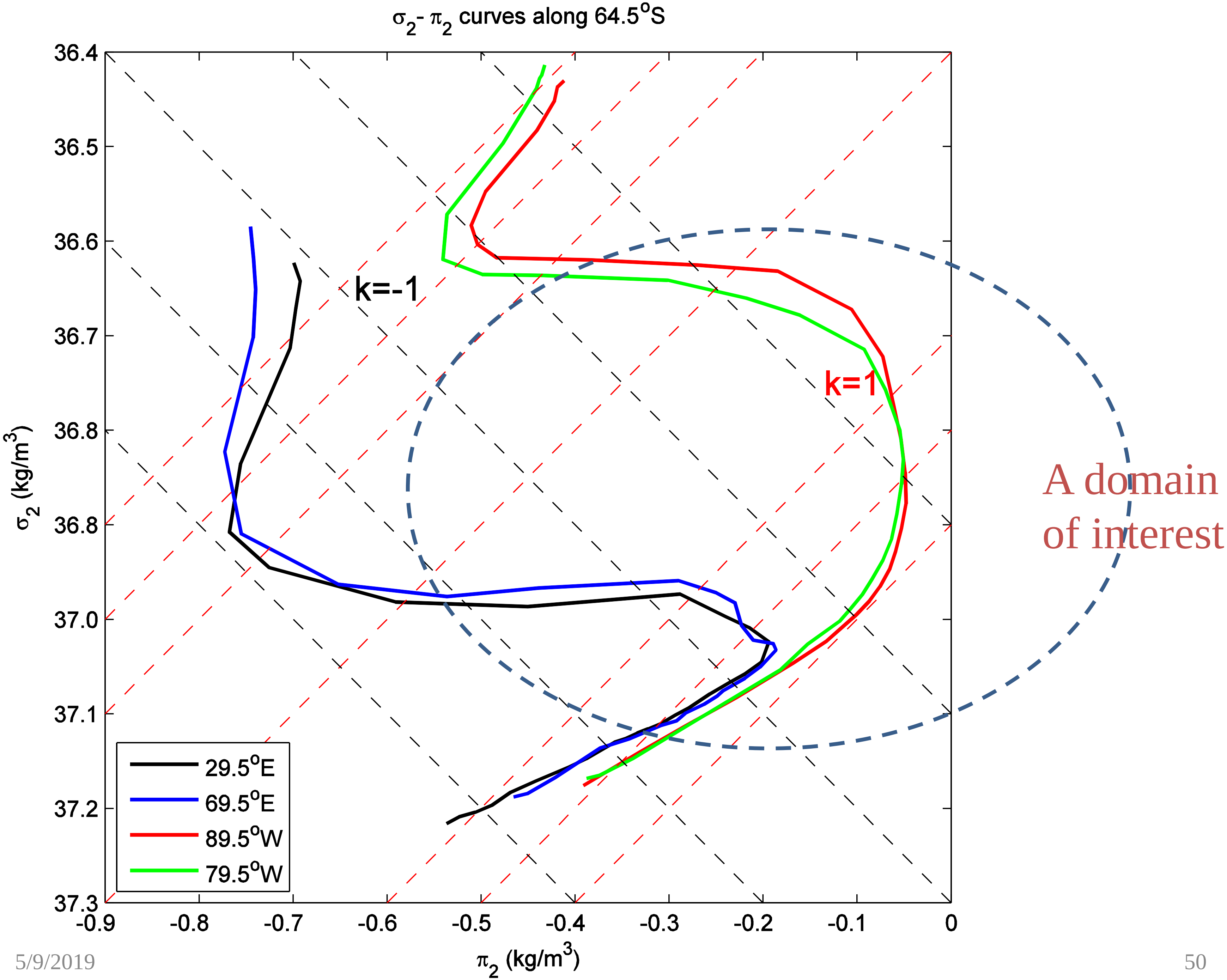
A comparison of T-S and s-p diagram for the recent double diffusion observation

(Schmitt et al., 2005)

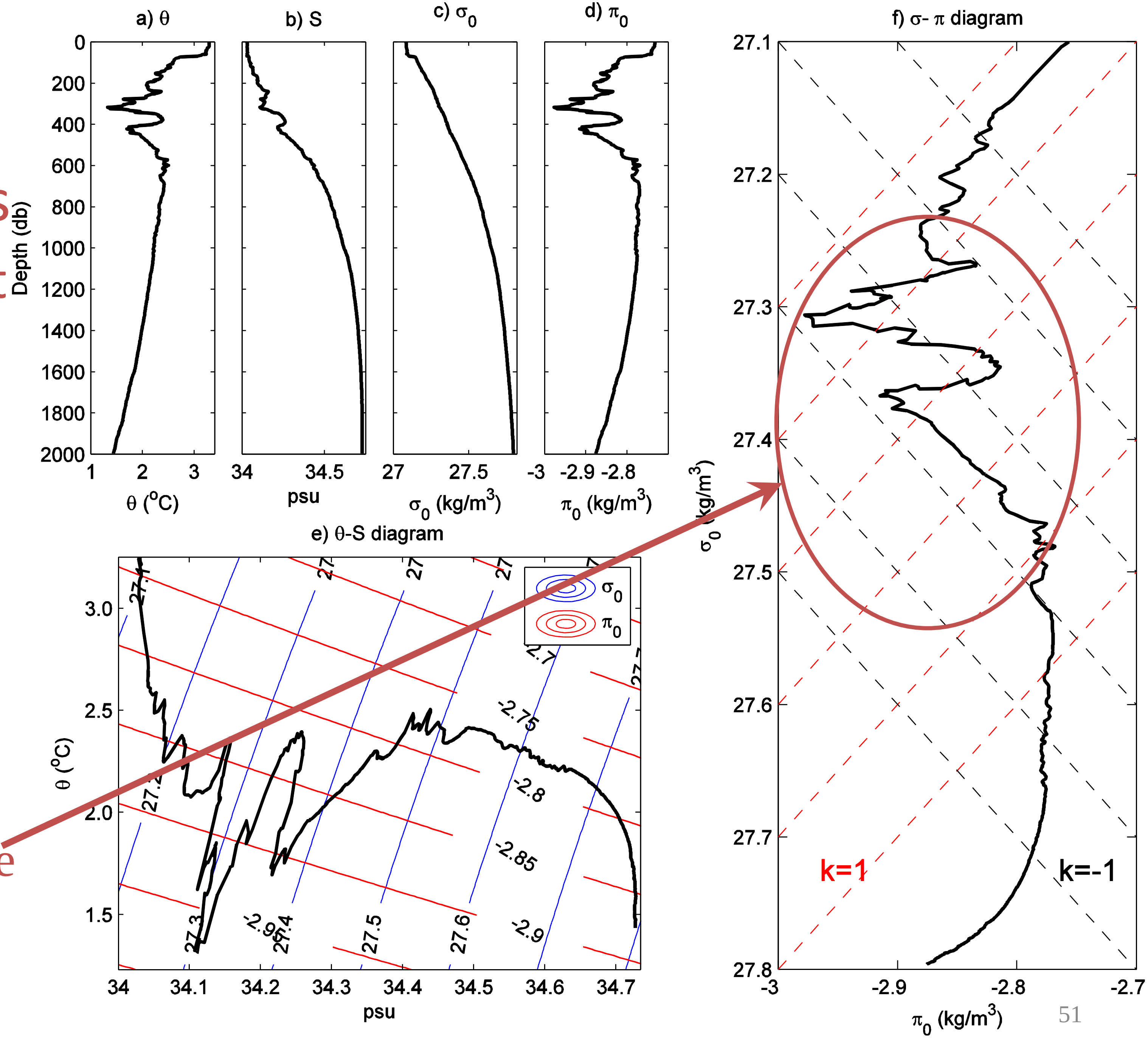


Water mass properties, 64.5°S




σ_2- π_2 curves along 64.5°S
36.4
36.5
36.6
36.7
36.8
36.8
37.0
37.1
37.2
37.3
σ_2 (kg/m^3)
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
π_2 (kg/m^3)
k=-1
k=1
29.5°E
69.5°E
89.5°W
79.5°W
A domain
of interest

Water mass properties, Argo Float 7900676, Cycle #103 (106.277°W, 61.49°S)

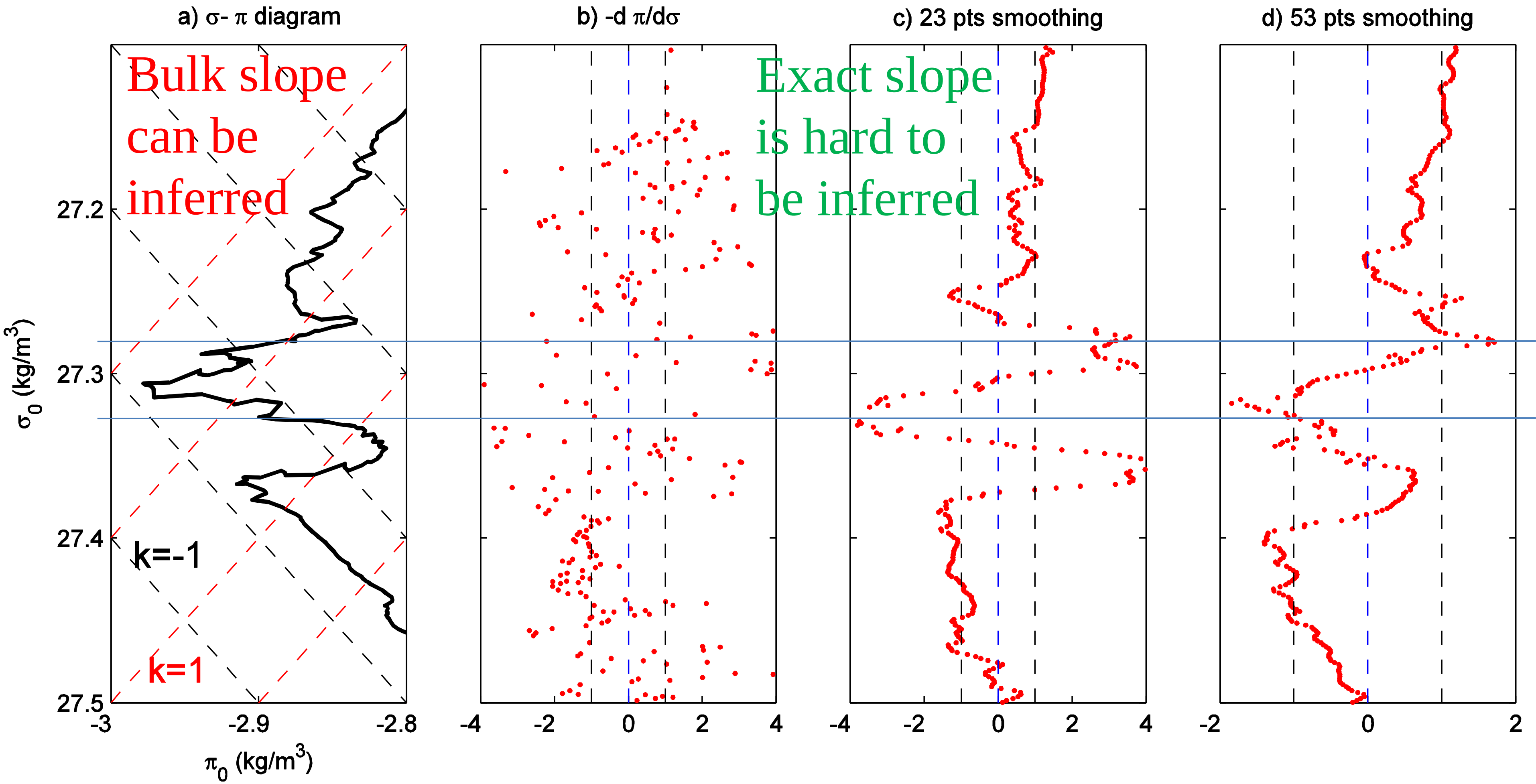


Thermohaline intrusion

Bulk slope can be directly inferred from s-p diagram

Slope directly calculated from the grid pairs in original data is too noisy

With 2-db resolution the data is rich in fine scale turbulence



Can we separate the signals?

- Modern CTD observations provide high resolution profile, with dp~1-2 db
- There are more than 200 millions ARGO profiles, plus many other high resolution observations.
- Can be separate signals from such huge amount of observations into

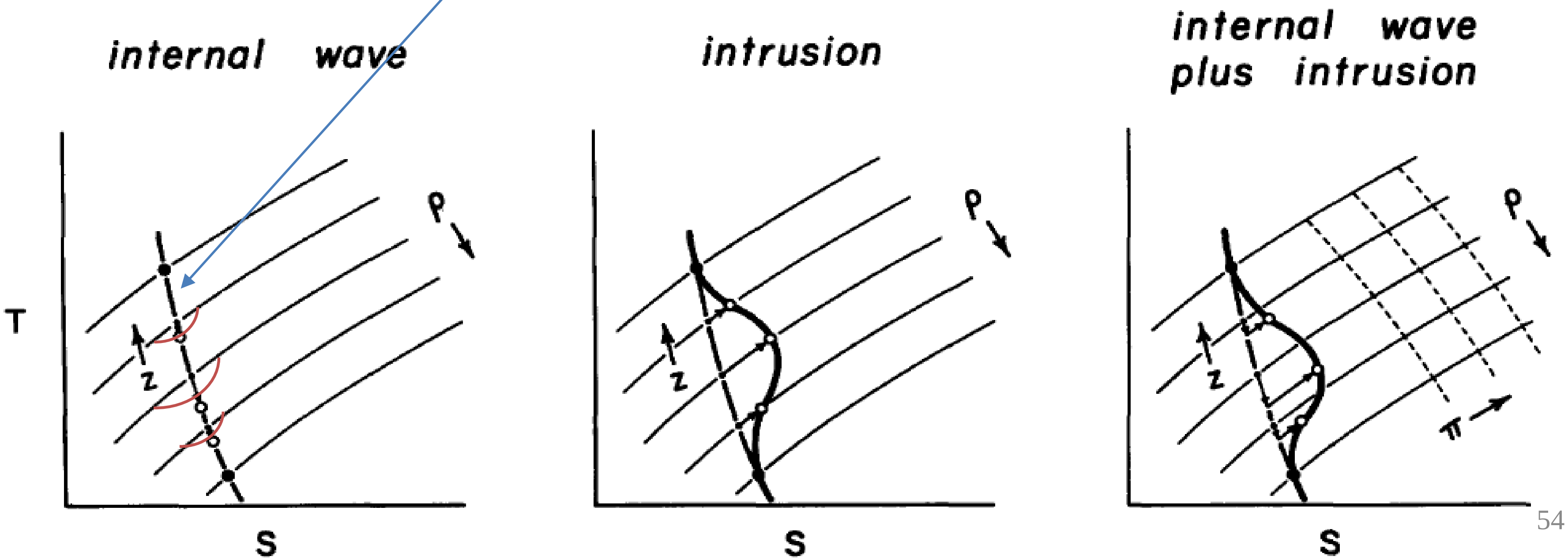
  turbulence (dz~ 1m); internal waves and thermohaline intrusion induced by currents and mesoscale and submesoscale eddies (dz~10 m -100m)

- This is the grand challenge for young scientists like you !!

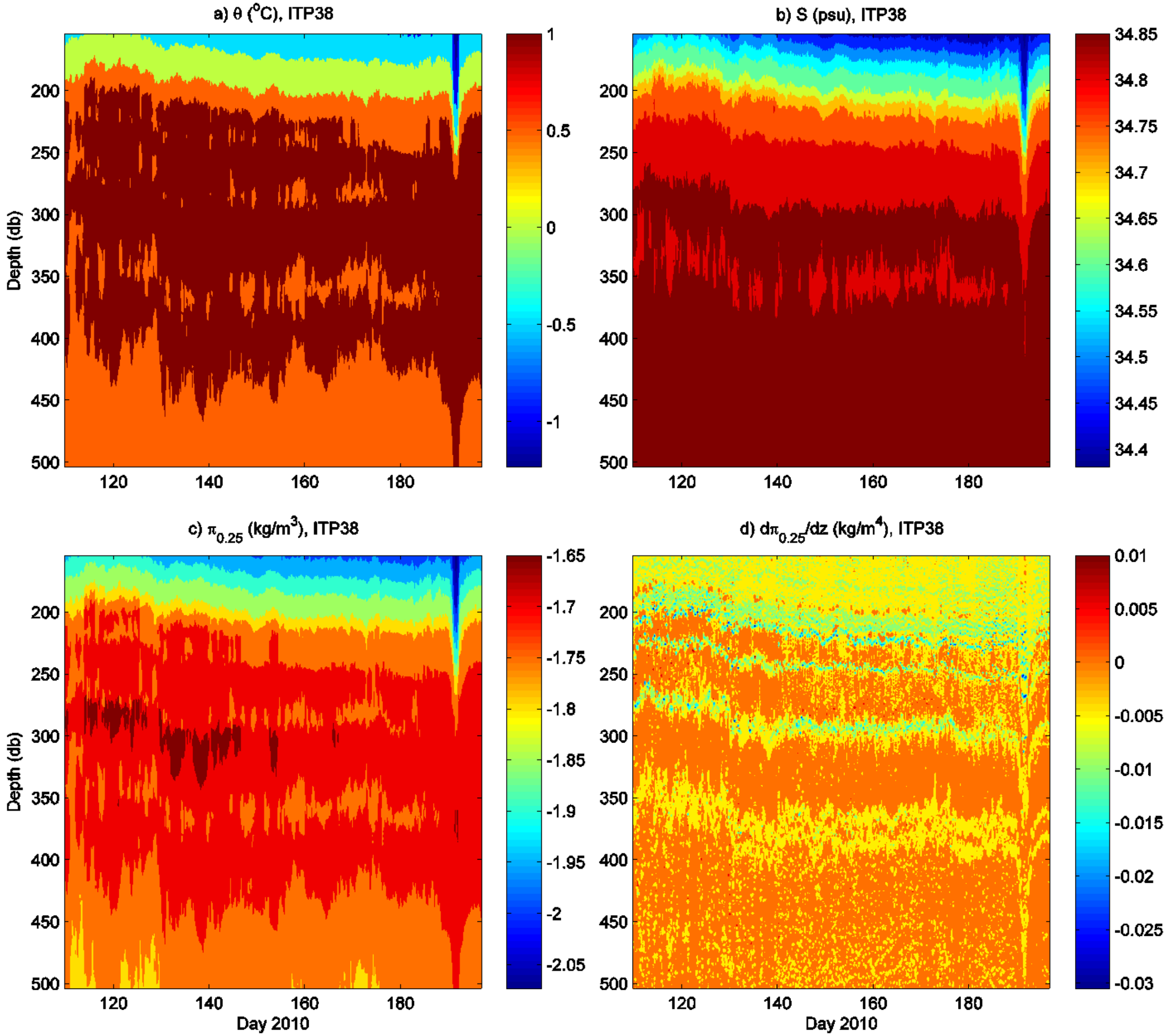
Munk (1980): "Internal waves and small-scale processes"

Internal waves and thermohaline intrusion can be clearly identified and quantified on the density-spiciness diagram

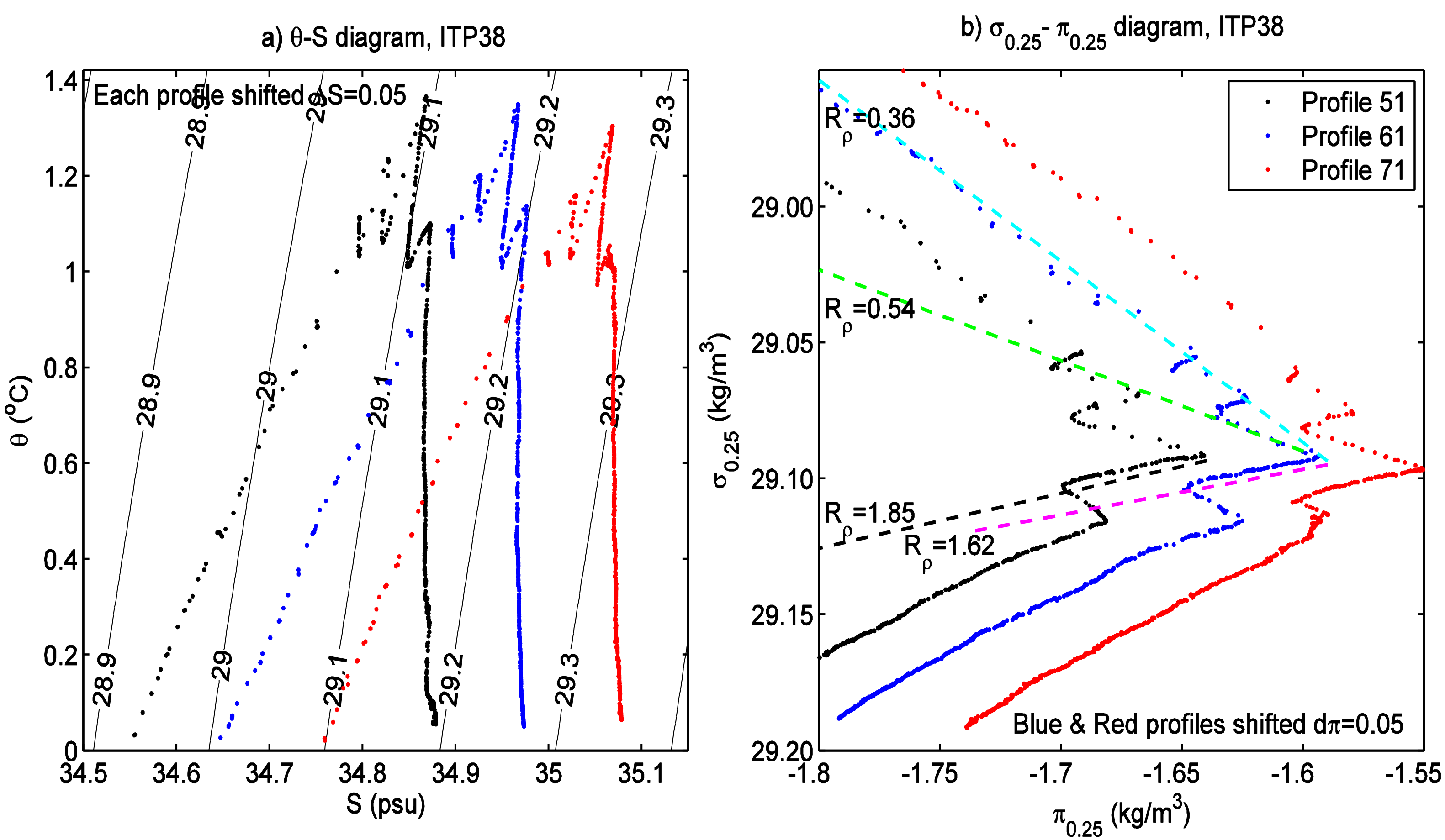
Caution: internal wave perturbations may not stay on the same T-S curve??



Water mass properties taken from ITP39

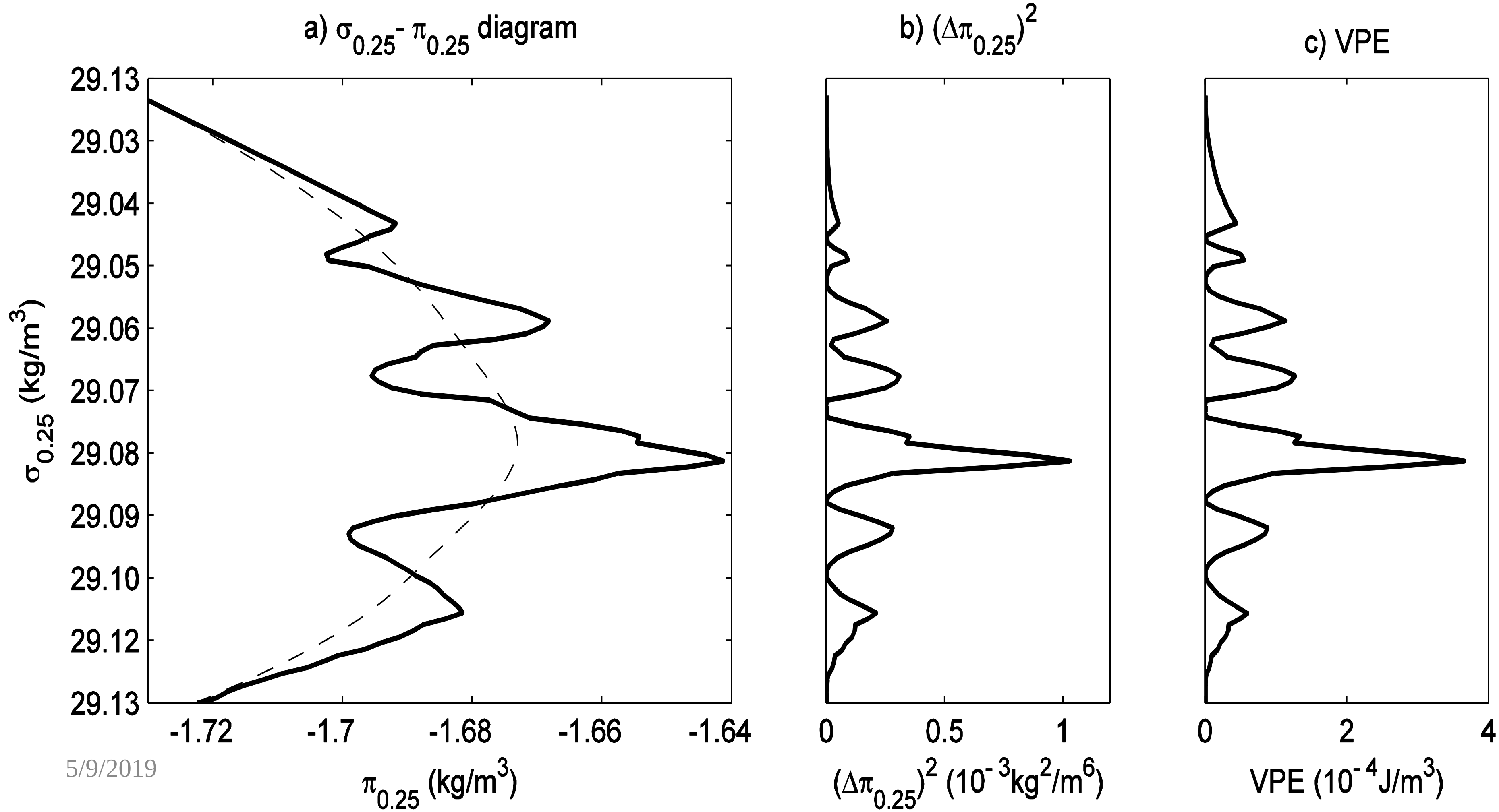


Thermohaline intrusion observed by ITP in the Arctic Ocean



Application of a segment of ITP profile

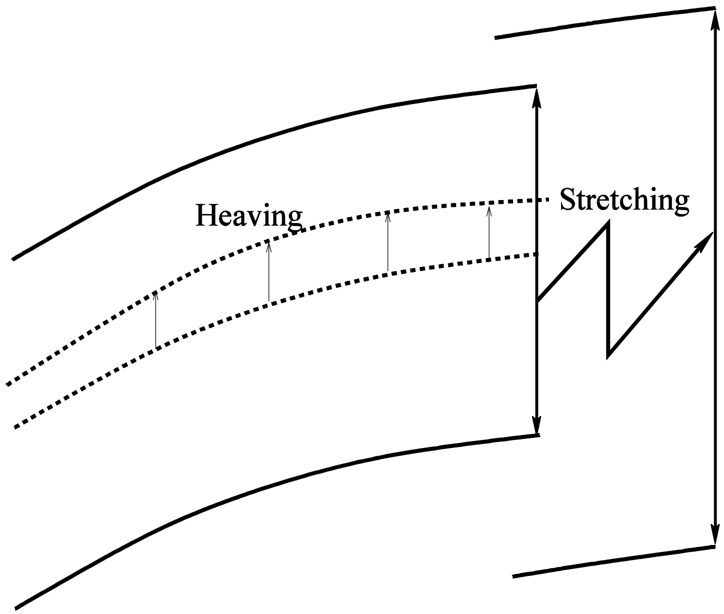
Data smoothing leads to useful quantifiable information



A tool for climate analysis:

Isopycnal layer analysis

Heaving, stretching and spicing modes



Movement associated with isopycnal layer

Spicity change due to advection, mixing, spiciling

$$\frac{\partial}{\partial t}(h\pi) + \nabla_\rho \cdot \left(h\vec{u}\pi\right) = \nabla_\rho \cdot \left[k_\rho \nabla_\rho (h\pi)\right] + \mathcal{H}_\pi - \frac{\partial}{\partial \rho}(\dot{\rho} h\pi)$$

Contributions due to heaving, stretching and spicing

For an equatorial station