A Practical Separation of Oceanic Vortical and Wavy Motions Entangled in the SWOT Measurements

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Key Points:

- A dynamics-based method is used to conduct a practical separation of vortical and wavy motions in SWOT measurements
- This separation method is based on the linear normal-mode initialization technique used in numerical weather prediction

Abstract

The recently launched Surface Water and Ocean Topography (SWOT) satellite provides an unprecedented two-dimensional measurement of the sea surface height down to the oceanic submesoscale of 1–10 km. Using this measurement to make substantial progress requires the separation of vortical and wavy motions owing to their contrasting ramifications for the energy transfer; however, the separation is extremely challenging due to the long-repeat period of the SWOT satellite. To achieve a practical separation, here we adopt the linear normal-mode initialization technique used in numerical weather prediction. This separation method requires velocity data in addition to sea surface height. With concurrent measurements of sea surface height and velocity respectively from SWOT and the offshore high-frequency radar system, this separation method proves valid and useful. The present study is expected to stimulate new discoveries associated with oceanic multiscale interactions and energy transfers.

Plain Language Summary

The Surface Water and Ocean Topography (SWOT) satellite measures a wide area of the sea surface height. These measurements contain two types of oceanic motions. One is the vortical motion generally including the mesoscale flow and submesoscale processes. The other is the wavy motion mainly incorporating the (internal) inertial-gravity wave. In SWOT measurements, one type of motion could hide the other. For example, the wavy (vortical) signal might become invisible in the oceanic region characterized by the intense vortical (wavy) motion. This raises the need to separate the two types of motions from SWOT measurements, which remains a major challenge due to their overlap in horizontal scales. To this end, this study uses the initialization technique in numerical weather prediction and realizes a practical vortical-wavy separation of SWOT measurements via additionally introducing concurrent sea surface velocity data from the land-based high-frequency radar system. The applications highlight the validity and usefulness of this separation approach.

1 Introduction

The Surface Water and Ocean Topography (SWOT) satellite, launched in December 2022, provides the first two-dimensional measurement of sea surface height (SSH) with an unprecedented spatial resolution of O(10 km) (Fu et al., 2024). Compared with the conventional nadir-looking altimetry, the SWOT satellite has the unique advantage of accurately measuring barotropic tides in complex coastal regions (Hart-Davis et al., 2024), internal gravity waves (Archer et al., 2025; Qiu et al., 2024) and submesoscale processes (Archer et al., 2025; Zhang et al., 2024). As a result, the measured SSH by SWOT contains combined contributions of vortical (including large-scale circulations, mesoscale eddies and submesoscale currents) and wavy (including barotropic tides and internal gravity waves) motions. It is of necessity to separate vortical and wavy motions due to their contrasting impact on oceanic energy transfer and turbulent mixing (Klein et al., 2019); a proper removal of wavy signals is also necessary to achieve an accurate estimation of geostrophic velocities from the SWOT measured SSH (Morrow et al., 2023). However, such a separation is very challenging owing to the long-repeat period (i.e., 1 day during the CalVal phase and 21 days during the science phase) of the SWOT satellite which inadequately captures the temporal evolution of submesoscale currents and internal gravity waves (Le Guillou et al., 2021; Klein et al., 2019; Morrow et al., 2019).

Several attempts have been made to address this challenge. One is to exploit the temporal aliasing caused by the long-repeat period to recover coherent internal tides. This approach has long been applied to conventional, nadir-looking satellite altimetric data (Carrère et al., 2004; Dushaw, 2015; Kantha & Tierney, 1997; Ray & Zaron, 2016; Ray & Cartwright, 2001; Ray & Mitchum, 1996; Zaron, 2019; Zhao, 2017; Zhao et al., 2016) and was recently applied to SWOT measurements (Tchilibou et al., 2024). However, this approach does not apply to the extraction of incoherent internal tides and internal gravity waves at non-tidal frequencies. Another approach, which only works for the summertime, employs the spatial filtering to separate vortical and wavy SSH with the cutoff chosen as the slope discontinuity of SSH wavenumber spectra (Torres et al., 2019). An additional approach is through assimilating SWOT measured SSH into the numerical ocean model, the output of which is then used to separate vortical and wavy motions. Le Guillou et al. (2021) and Yadidya et al. (2024) provide preliminary application examples. More advanced techniques are required to tackle the difficulty of directly assimilating SWOT SSH which has multiscale characteristics. The machine learning also proves useful (Gao et al., 2024; Lguensat et al., 2020; H. Wang et al., 2022); nevertheless, its applicability to SWOT measurements remains unknown.

Here we hightlight that the initialization technique in numerical weather prediction can inspire a more general and accurate vortical-wavy separation that is applicable to SWOT measurements. In the history of weather prediction using primitive equations models, there exists a long-standing effort to reduce or arrest the growth of meteorologically-unimportant internal gravity waves via defining a balance between the initial pressure and velocity fields (e.g., Coiffier, 2011). An adequately successful attempt is the linear normal-mode initialization (Dickinson & Williamson, 1972; Williamson, 1976). Building on the fact that vortical and wavy modes are eigenfunctions of the linearized governing equations, this initialization filters out internal gravity waves via directly setting the wavy mode in the initial fields to be zero. This filtering is exactly consistent with the fundamental property that wavy motions do not induce any potential vorticity (PV) anomaly relative to the rest state (Pedlosky, 2003; Zeitlin, 2018). For this reason, this initialization technique is called the PV-based method in this study. Recently, C. Wang et al. (2025) formulated the PV-based method in the rotating shallow water system and made a proof-of-concept application to a concurrent snapshot (i.e., ~5° × 5° box) of sea surface height and velocity (SSV) extracted from a high-resolution numerical simulation. They show that the PV-based method is capable of achieving a satisfactory vortical-wavy separation in contrasting dynamical regimes (i.e., the South China Sea with strong internal tides but weak eddy activities and the Kuroshio Extension with strong eddy activities but weak internal tides). As a follow-up, this study applies the PV-based method to SWOT measurements of the real ocean.

For realistic application, the SSV measurement concurrent with SWOT SSH is required. At the present time, this requirement is feasible for many parts of the coastal oceans where SSV from the high-frequency radar (HFR) system is available on SWOT swaths. The HFR data have shown an encouraging capability in capturing submesoscale processes (Chavanne et al., 2010; Lai et al., 2017; Payandeh et al., 2023; Soh & Kim, 2018; Yoo et al., 2018) and internal tides (Kachelein et al., 2024; Lee & Kim, 2022). Here we choose the offshore region of California, which is well supported with a HFR network, to test the PV-based method. We will proceed in two steps. Firstly, SSH and SSV from a realistic tide-resolving and submesoscale-admiting numerical simulation (i.e., MITgcm LLC4320) are regridded onto the swath-style grid of SWOT to mimic the real-ocean observations; then the PV-based method is applied to the regridded SSH and SSV and is validated against the baseline separation that will be described in Section 2.2. Secondly, the SWOT measured SSH and HFR measured SSV are remapped onto the SWOT grid and then vortical and wavy motions are separated.

2 Materials and Methods

2.1 SSH and SSV data

The SWOT satellite observes SSH over two parallel 50-km swaths interleaved with a 20-km nadir gap. The horizontal resolution of the SWOT SSH product, namely SWOT Level-3 (L3) SSH Expert, is 2 km over each swath. The HFR system routinely provides a two-dimensional measurement of SSV with a horizontal resolution of 6 km. In this study, we select an oceanic region (i.e., 35°-40°N, 235°-240°E) offshore of California since this region is well covered by the HFR system, making it easy to match SWOT observations. For illustrative convenience, we focus on a pair of SWOT SSH (Figure S1a in Supporting Information) and HFR SSV (Figures S1b-S1c) at ~2023-09-11 17:30:00. Figures S1d-S1f show SSH and SSV which are remapped onto SWOT swaths.

Prior to the application of the PV-based method to the real-ocean observations, we employ modelled SSH and SSV from the MITgcm LLC4320 simulation as a testbed. This global simulation has a horizontal grid spacing of ~2 km and outputs hourly snapshot variables from September 2011 to November 2012. In this study, we use the hourly model output offshore of California during September 2012. Figures S2a-S2c display the simulated SSH and SSV at 2012-09-11 17:30:00; SSH and SSV at the same time, which are remapped onto SWOT swaths, are shown in Figures S2d-S2f. More details about the LLC4320 simulation can be found in Arbic et al. (2018).

2.2 The PV-based separation

In the following, we concisely describe the mathematical formulations of the PV-based method for a rotating shallow water system; for further details, we refer to Zeitlin (2018) and C. Wang et al. (2025). That is,

$$\nabla^2\left(\frac{g\bar{\eta}}{f_0}\right)-\frac{1}{L_d^2}\frac{g\bar{\eta}}{f_0}=\zeta-\frac{1}{L_d^2}\frac{g\eta}{f_0} \tag{1}$$

$$\boldsymbol{f_0}\times\bar{\boldsymbol{u}}=-g\nabla\bar{\eta} \tag{2}$$

$$\eta'=\eta-\bar{\eta} \tag{3}$$

$$\boldsymbol{u}'=\boldsymbol{u}-\bar{\boldsymbol{u}} \tag{4}$$

where the overbar represents the vortical variable, the prime the wavy variable, $\boldsymbol{u}=(u,v)$ SSV, η SSH, $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ the vertical component of the relative vorticity (hereafter referred simply to as the relative vorticity), g the acceleration due to gravity, $\boldsymbol{f_0}=(0,0,f_0)$ with f_0 being the Coriolis parameter and L_d the deformation radius. It is emphasized that we adopt a pragmatic manner to define L_d as the effective deformation radius (Figure S3) which considers contributions of all vertical modes; detailed introductions to the determination of L_d can be found in Section S1 of Supporting Information. Due to the peculiar domain geometry of SWOT data, the procedures for solving equations (1-4) in the present study are quite different from those in C. Wang et al. (2025); see Section S1 for details. We also use the decomposition approach of C. Wang et al. (2023a) to obtain baseline vortical and wavy variables for the LLC4320 simulation and validate the PV-based separation results against those baseline truth; more information about the baseline separation is given in Section S2 where Lagrangian filtering (Shakespeare et al., 2021) is additionally introduced to demonstrate that the decomposition approach of C. Wang et al. (2023a) well serves as the baseline.

3 Results

3.1 Decomposed variables using the numerical model data

In this section, we apply the PV-based method to the LLC4320 simulation. The separated wavy SSH, zonal velocity and meridional velocity are shown in Figures 1a-1c, respectively. Due to the strong incoherence of baroclinic tides offshore of California (Kachelein et al., 2024), wavy SSH and SSV are dominated by irregular spatial patterns. Those irregular features essentially follow the dispersion relation curves of internal gravity waves and tidal frequencies in the frequency-wavenumber spectra (Figures S4a and S4d) which are calculated after the PV-based method is applied to each snapshot of SSH and SSV. Compared with wavy SSH and SSV, the irregular patterns in wavy horizontal divergence (Figure 2a) and wavy relative vorticity (Figure 2b) are characterized by much finer horizontal scales since the spatial differentiation tends to amplify the contributions from high-wavenumber features (C. Wang et al., 2023b). Constrained by the relative vorticity equation in spectral space (i.e., equation (5) below) of internal gravity waves whose frequency is larger than f_0, the magnitude of wavy horizontal divergence (solid red line in Figure 3) is generally larger than that of wavy relative vorticity (solid blue line) and such magnitude difference generally becomes more pronounced with the increasing wavenumber.

$$i\omega\widehat{\zeta'} + f_0\widehat{\chi'} = 0 \quad (5)$$

where the caret (^) denotes Fourier-transformed variables, χ is the horizontal divergence and $\omega^2 = c_e^2 K^2 + f_0^2$ with K representing the isotropic wavenumber and c_e the phase speed corresponding to L_d defined in Section S1. Using equation (5), we can derive wavy horizontal divergence from wavy relative vorticity; it is found that although there exists a magnitude difference between the target (solid red line in Figure 3) and derived (solid magenta line) divergence wavenumber spectra, variations of the spectra with the increasing wavenumber seem to be quite consistent. This tends to suggest the dynamical consistency among wavy vorticity and divergence. The consistency is re-confirmed by the joint probability distribution function patterns of $\zeta' - \chi'$ in Figure S5a and $\zeta' - \sigma'$ (σ is the strain) in Figure S5b (Xiao et al., 2023). To further quantitatively assess how well the PV-based method works, we validate the separated wavy variables against the baseline truth. For wavy SSH (SSV), the root mean square errors (Figures S6a-S6c) between the PV-based and baseline results are generally smaller than 0.008 m (0.06 m/s) over the entire domain and negligible compared with its typical magnitude (i.e., 0.04 for wavy SSH and 0.4 for wavy SSV in Figures 1a-1c); the correlation coefficients (Figures S7a-S7c) are above 0.8 over almost the whole study region. The small root mean square errors and high correlations suggest that wavy motions derived from the PV-based method agree well with the baseline in terms of the magnitude, spatial pattern and temporal evolution.

The vortical SSH, zonal velocity and meridional velocity are displayed in Figures 1d-1f , respectively. Both mesoscale and submesoscale features are revealed. Submesoscale vorticity filaments are particularly clear in Figure 2c. The joint probability distribution function pattern of $\bar{\zeta} - \bar{\sigma}$ in Figure S5c is representative of vortical motions (Rocha et al., 2016; Shcherbina et al., 2013; Xiao et al., 2023). The horizontal divergence vanishes (not shown) since vortical SSH and SSV are in geostrophic balance by construction. The wavenumber-frequency spectra in Figures S4b-S4e show that vortical motions are mostly sub-inertial. Although there exists some spectral energy leakage at the tidal and near-inertial frequencies, the extraction of vortical motions is at least satisfactory (Figures S4c-S4f). The quantitative comparision with the baseline vortical variables, in terms of small root mean square errors in Figures S6d-S6f and high correlation in Figures S7d-S7f, demonstrates a favorable agreement.

Overall, the PV-based method proves satisfactorily applicable to pseudo-SWOT data based on the LLC4320 simulation. In the next section, we apply this separation method to the real-ocean observational data.

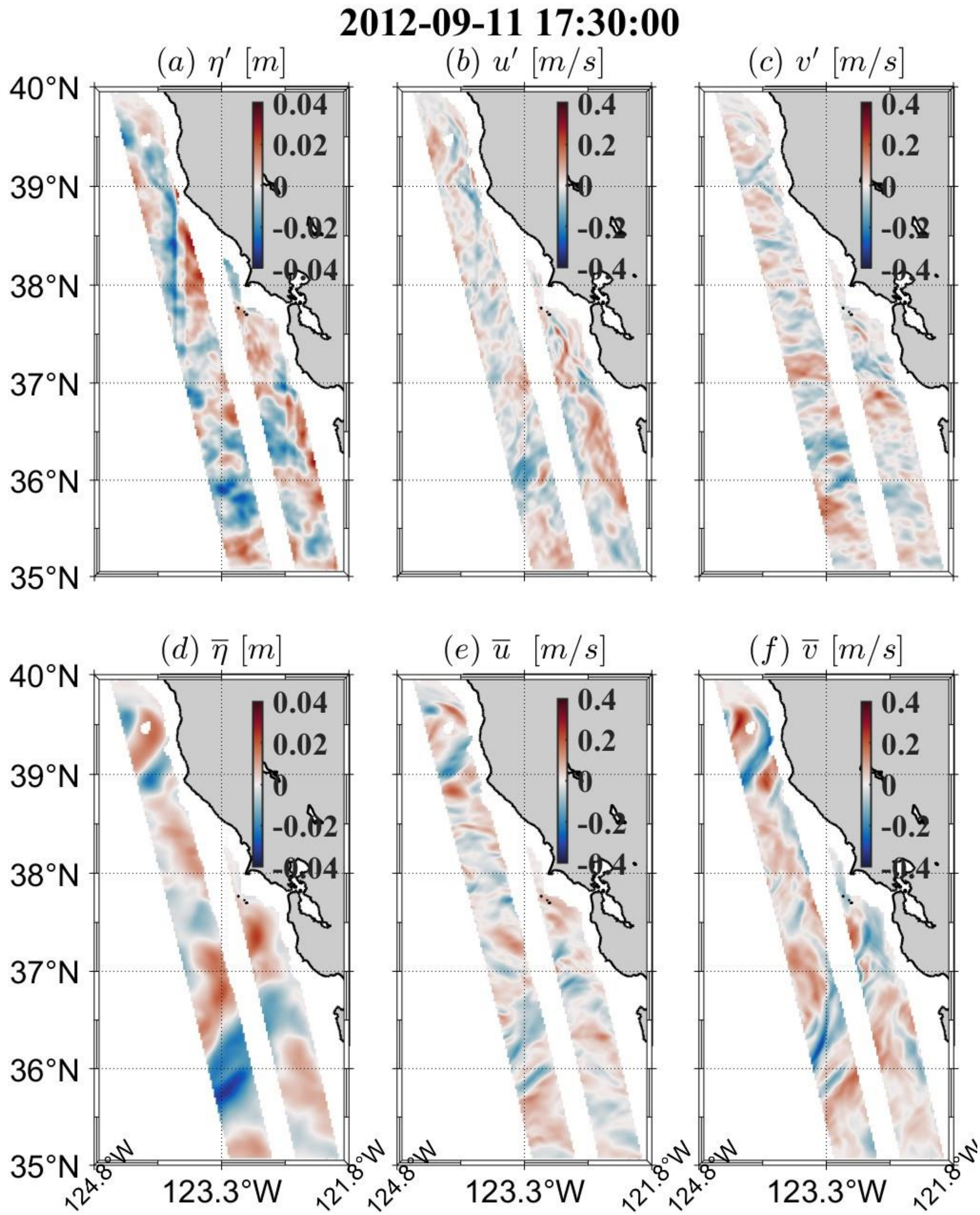

Figure 1. The decomposed SSH (a, d), zonal velocity (b, e) and meridional velocity (c, f) based on the LLC4320 simulation offshore of California. The upper (a-c) and lower (d-f) panels represent wavy and vortical motions, respectively.

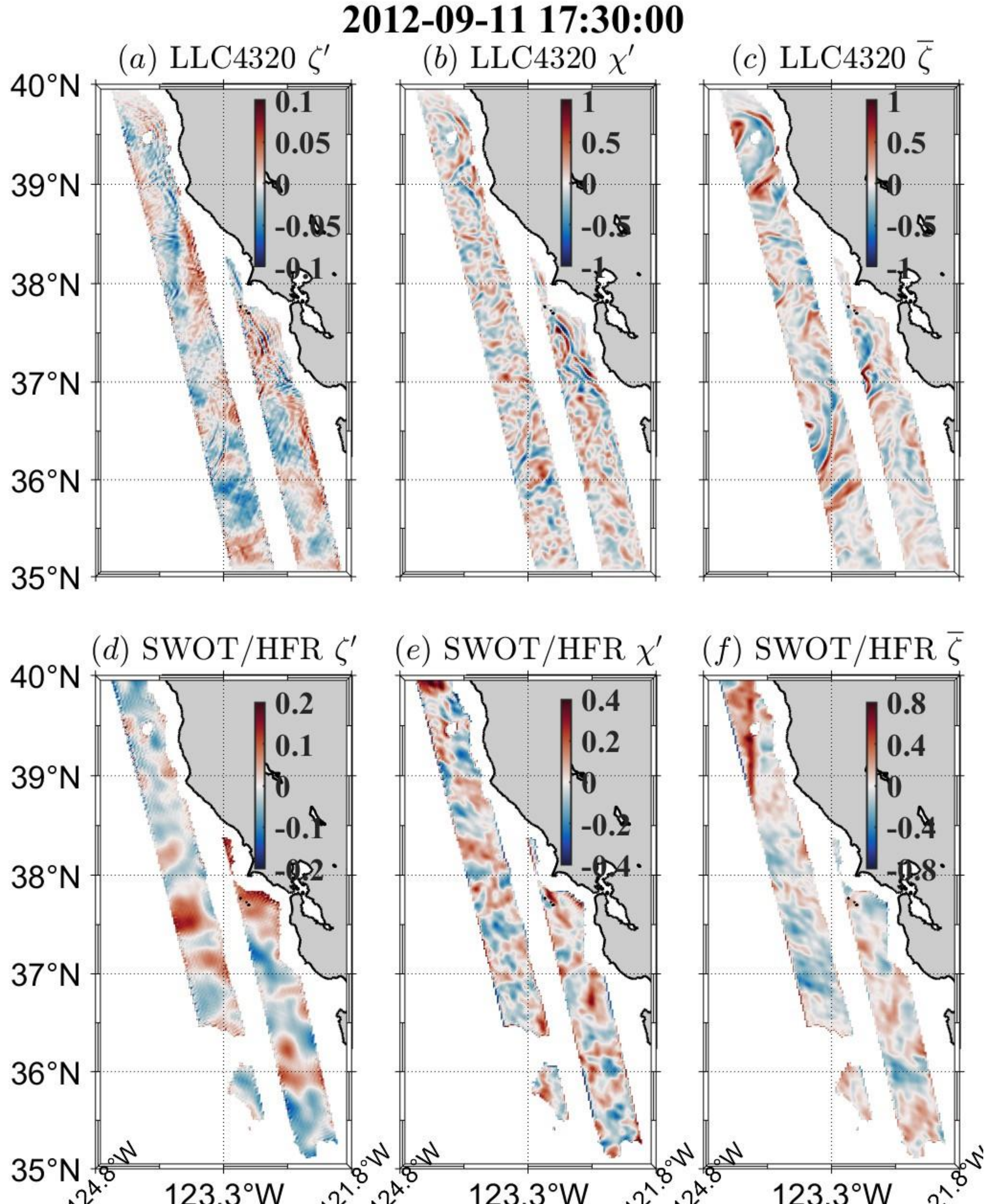

Figure 2. The decomposed relative vorticity (a, c, d, f) and horizontal divergence (b, e) based on LLC4320 (a, b, c) and SWOT/HFR (d, e, f) offshore of California. The first two columns (a, b, d,

e) and third (c, f) column represent wavy and vortical motions, respectively. Note the different color scales between LLC4320 and SWOT/HFR.

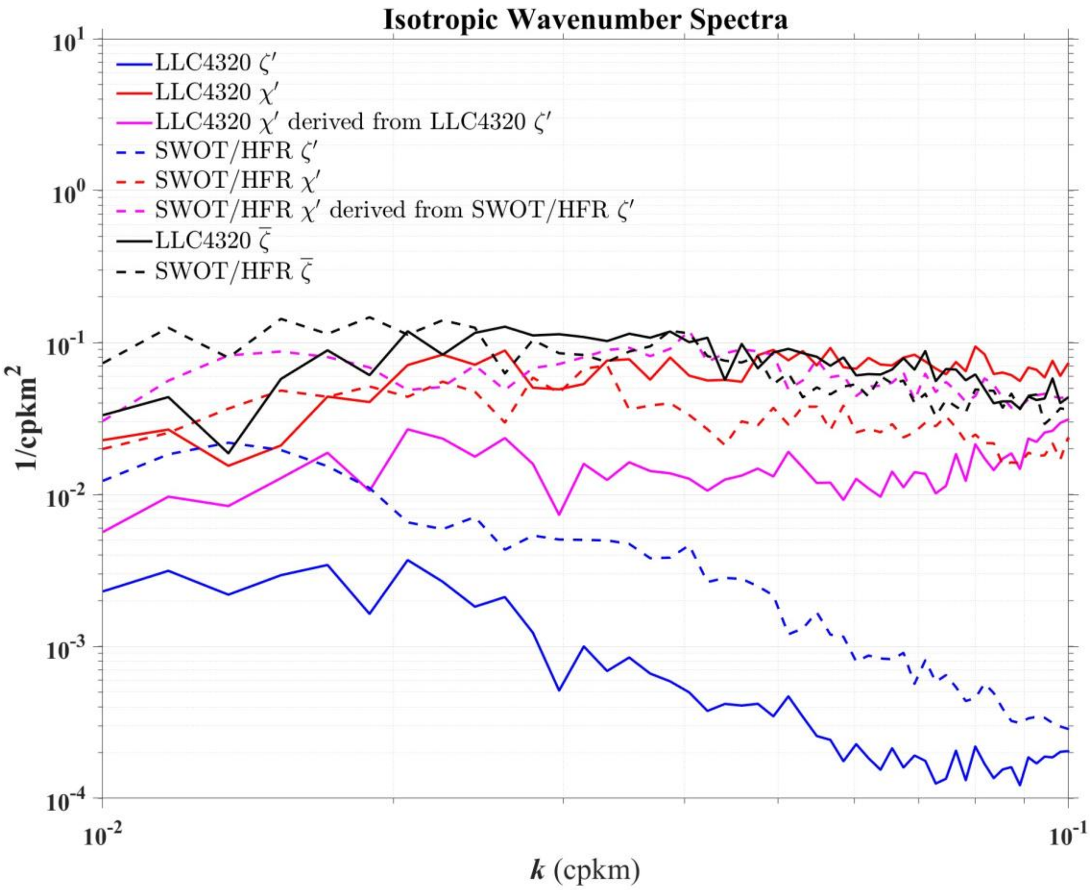

Figure 3. The isotropic wavenumber spectra for wavy and vortical motions. The solid and dashed lines are based on LLC4320 and SWOT/HFR, respectively. The blue, red and black lines show wavy relative vorticity, wavy horizontal divergence and vortical relative vorticity, respectively. The magenta lines represent wavy horizontal divergence derived from wavy relative vorticity. Note that the plot is cut at 10^{-2} cpkm and 10^{-1} cpkm to remove the artificial effect of zero-filling.

3.2 Decomposed variables using SWOT/HFR measurements

Figure 4 shows vortical and wavy SSH and SSV separated from SWOT/HFR measurements. The corresponding horizontal divergence and relative vorticity are shown in Figures 2d-2f. The irregular spatial distributions of wavy variables (Figures 4a-4c and Figures 2d-2e) indicate the dominant contribution of incoherent baroclinic tides offshore of California, which has been recently confirmed by HFR data (Kachelein et al., 2024). As explained in Section 3.1, the wave dynamics requires that wavy relative vorticity (Figure 2d) is dominated by a larger horizontal scale but a smaller magnitude than wavy horizontal divergence (Figure 2e); again, this magnitude distinction becomes clearer as the wavenumber increases, as shown by the dashed blue and red lines in Figure 3. The spectra of the derived wavy horizontal divergence (dashed magenta line in Figure 3) expectedly follow that of the target one (dashed red line). It is interesting to note that compared with SWOT/HFR, the LLC4320 simulation well reproduces the spectral energy level of wavy horizontal divergence but underestimates that of wavy relative vorticity. These results appear to agree with the previous finding that LLC4320 lies close to McLane profiling observations in the supertidal band (Savage et al., 2017) which is more dominated by horizontal divergence than relative vorticity. For vortical variables (Figures 4d-4f and Figure 2f), abundant mesoscale features in geostrophic balance and without horizontal divergence are revealed. The submesoscale processes with high relative vorticity are also identifiable, especially in Figure 2f. However, filamentary structures present in LLC4320 (i.e., Figure 2c) are lacking in SWOT/HFR (i.e., Figure 2f); there are many potential reasons but one could be the spatial resolution contrast (i.e., 6 km versus 2 km) between the HFR and LLC4320 data. Otherwise, LLC4320 and SWOT/HFR vortical relative vorticity fields have the same energy across all scales as shown by solid and dashed black lines in Figure 3; this highlights the usefulness of the high-resolution simulation in interpreting SWOT/HFR data. Comparatively, vortical and wavy variables in this oceanic region have the same magnitude. As a result, wavy SSH could distort or overwhelm vortical SSH, re-emphasizing the importance of removing the wavy signal prior to utilizing SWOT SSH for (sub)mesoscale-related studies. Therefore, it is informative to compare the vortical SSH extracted from SWOT data with the nadir-looking altimetric SSH (i.e., AVISO SSH). For comparison, we use the nearest-neighbor interpolation to remap AVISO SSH onto the finer grid of SWOT vortical SSH since this interpolation method does not introduce artificial submesoscale features. It is found that after the removal of the wavy signal, the general pattern of SWOT vortical SSH (Figure 4d or Figure S8a) qualitatively agrees with that of AVISO SSH (Figure S8b). However, their quantitative difference reaches a non-negligible magnitude of ~0.04 m (Figure S8c); importantly, the difference contains both mesoscale (i.e., pixel scale in Figure S8c) and submesoscale (i.e., sub-pixel scale in Figure S8c) features. This indicates that the SWOT satellite not only improves the accuracy of observing mesoscale eddies but also achieves the intention of capturing submesocale currents.

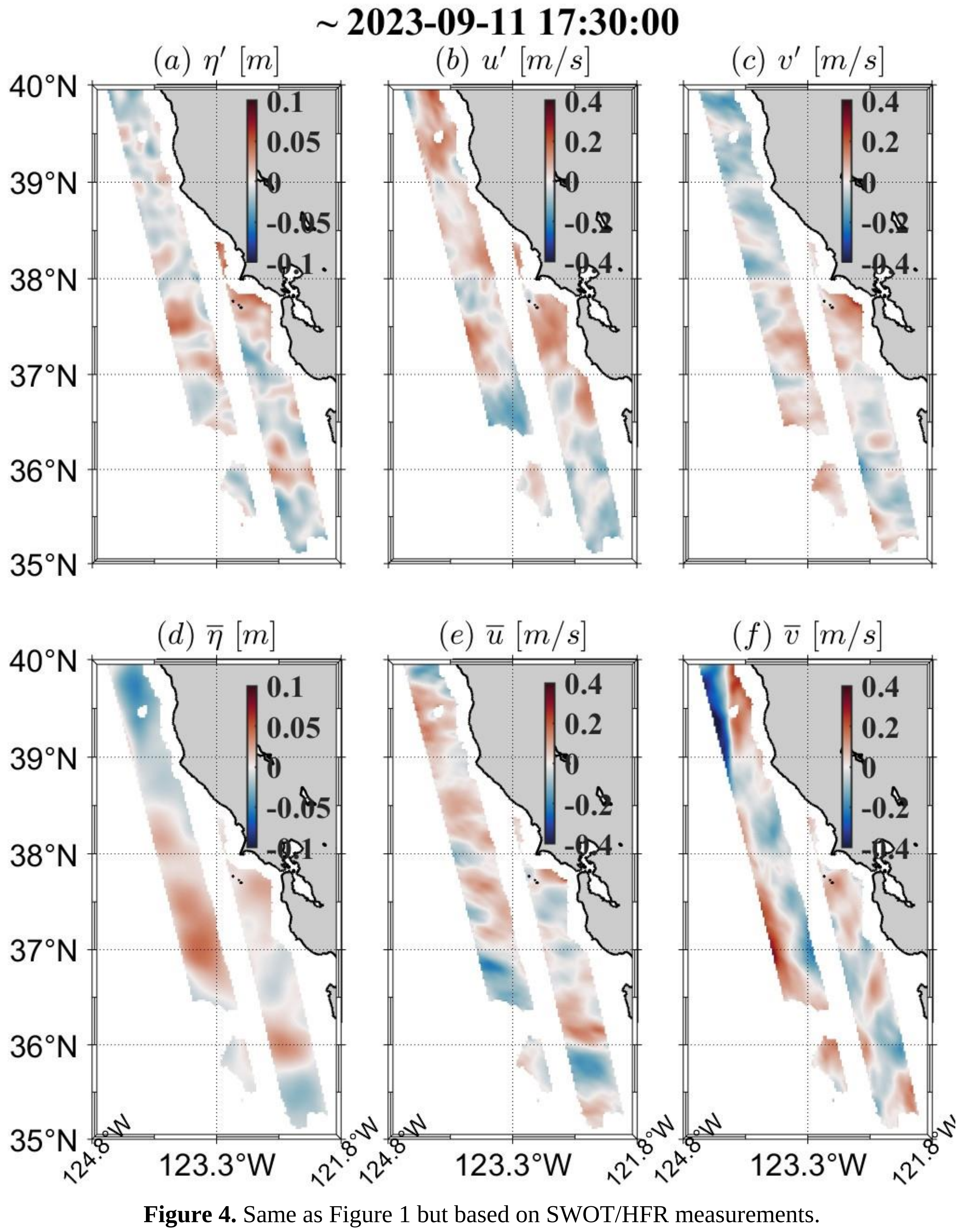

Figure 4. Same as Figure 1 but based on SWOT/HFR measurements.

4 Summary and Discussion

Realizing the concurrent availability of HFR SSV and SWOT SSH, we use a PV-based method to address the challenge of separating vortical and wavy motions intermingled in those observations. This PV-based separation is exactly the linear normal mode initialization which has played an important role in the history of numerical weather prediction. When applied to concurrent SSH and SSV extracted from the LLC4320 simulation and remapped onto SWOT swaths offshore of California, the PV-based method shows good performance compared with the baseline truth. The performance supports the utility of this method to swath-style data and motivates its application to real-ocean observations. With SSH and SSV respectively from the SWOT satellite and the HFR system offshore of California, the separated results confirm the capability of the SWOT satellite to capture submesoscale processes, highlight the necessity of removing the wavy signal before conducting (sub)mesoscale-oriented studies and reveal the general pattern agreement between vortical SSH observed by SWOT and SSH by the conventional satellite altimeter.

Since separating vortical and wavy motions in SWOT measurements is a challenging difficulty, we regard this study as a first-step attempt towards a more accurate separation; therefore, it is necessary to clearly discuss the limitations of the present study and potential improvements in future studies.

Firstly, the present study particularly applies to the regime where 1) Rossby and Froude numbers are small and 2) vortical and wavy motions have comparable magnitude. Beyond that parameter regime, the theoretical basis of this study that wavy motions carry no PV anomaly breaks down. For example, the stronger internal gravity waves can non-negligibly modulate the PV anomaly of the weaker vortical motions (Bühler & McIntyre, 1998; Rocha et al., 2018; Wagner & Young, 2015; Xie & Vanneste, 2015). The gravity-wave-induced forcing can even resonantly trigger Rossby waves (Bühler & McIntyre, 1998). More complicated initialization techniques, such as nonlinear normal mode initialization (Baer & Tribbia, 1977; Chouksey et al., 2018; Machenhauer, 1977), digital filtering (Lynch & Huang, 1992; Lynch et al., 1997), quasi-geostrophic theory with a next-order correction (Dù & Smith, 2024; Spall & McWilliams, 1992; Vallis, 1996; Warn et al., 1995), optimal PV balance (Viúdez & Dritschel, 2004) and optimal balance (Chouksey et al., 2023; Masur & Oliver, 2020; Rosenau et al., 2025), have the potential to consider most/all parameter regimes and merit a future pursuit.

Secondly, that the PV anomaly is zero is a necessary rather than sufficient condition for wavy motions. Typical examples of PV-free vortical motions include the surface quasi-geostrophic current in particular (Held et al., 1995; Guillaume Lapeyre, 2017) and the Eady-like flow in general (e.g., Callies et al., 2015; Molemaker et al., 2010). As can be seen in Section 2.2, equation (1) is exactly the interior quasi-geostrophic theory; consequently, vortical motions driven by the surface and/or bottom buoyancy anomaly might be misclassified into the wavy category. However, this misclassification might not pose a serious problem in the present study region with low mesoscale kinetic energies since buoyancy-driven vortical motions are most active in oceanic regions with intense mesoscale activities (Gonzalez-Haro & Isern-Fontanet, 2012; Gonzalez-Haro & Isern-Fontanet, 2014). To consider the boundary buoyancy effects and thus improve the vortical-wavy separation, it is necessary to additionally invoke the surface quasi-geostrophic theory which involves a third variable, namely sea surface density.

Thirdly, the main drawback of the PV-based method might be to assume that all vortical motions are in geostrophic balance and thus do not have horizontal divergence. This assumption can be problematic for submesoscale currents (Archer et al., 2025; Tranchant et al., 2025) and even for ~100-kilometer rings (Penven et al., 2014). The abovementioned advanced initialization techniques, which do not *a priori* assume the geostrophic balance, are capable of addressing this drawback.

Fourthly, given that our main goal is to test the usefulness of the PV-based method, we assume a rotating shallow water system with an effective deformation radius to simplify the challenging vortical-wavy separation. Combined with well-established subsurface reconstruction methods for (sub)mesoscale processes (Klein et al., 2009; LaCasce & Mahadevan, 2006; Lapeyre & Klein, 2006; Liu et al., 2019; Qiu et al., 2016; J. Wang et al., 2013) and internal gravity waves (Ray & Cartwright, 2001; Zhao, 2017; Zhao et al., 2016), the present study easily extends to the continuously stratified system.

Fifthly, we use the decomposition approach of C. Wang et al. (2023a) as the baseline. As mentioned in C. Wang et al. (2023a), this approach has some limitations. For example, the Gibbs phenomenon occurs due to the spectral cutoff characteristic of the 0/1-type filter; that vortical and wavy motions are mutually exclusive in spectral space is assumed. How these limitations affect the validation of the vortical-wavy separation in the present study remains unknown. We plan to pursue this in the future.

Finally, HFR data usually suffer from observational errors (Clary et al., 2019) whose adverse effects on the physical processes of interest remain to be explored in detail. In future studies, a realistic tide-resolving and submesoscale-admitting simulation simultaneous with SWOT and HFR observations could be made in order to quantify to what extent measurement limitations or inaccuracies contaminate the vortical-wavy separation.

Overall, this study suggests a promising research direction involving SWOT/HFR measurements. At the present time, the HFR system provides SSV observations over the coastal ocean; in the future, the Doppler scatterometric satellite will measure a wide swath of SSV over the global ocean (Du et al., 2021; Torres et al., 2023). Hopefully, this study would help understand multiscale ocean dynamics invigorated by those wide-swath satellite missions.

Acknowledgments

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Open Research

The SWOT Level-3 SSH Expert product is available at https://doi.org/10.24400/527896/A01-2023.018 (AVISO/DUACS, 2024). The high-frequency radar SSV data can be downloaded at https://doi.org/10.48670/moi-00041 (E.U. Copernicus Marine Service Information, 2024a). The nadir-looking satellite altimeter data are available from https://doi.org/10.48670/moi-00148 (E.U. Copernicus Marine Service Information, 2024b). The model output of the LLC4320 simulation can be accessed from https://data.nas.nasa.gov/ecco/data.php?dir=/eccodata/llc_4320 (ECCO

Consortium, 2025). The PV-based separation code adapted for SWOT and HFR data is accessible at https://doi.org/10.5281/zenodo.14088311 (Wang, 2024).

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