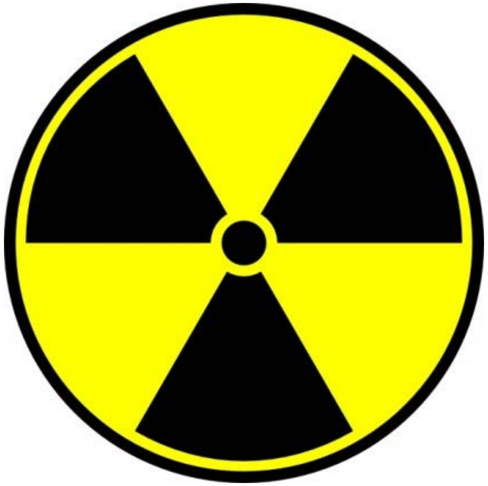
Dr. Claudia Benitez-Nelson

University of South Carolina
cbnelson@geol.sc.edu

UNIVERSITY OF
SOUTH CAROLINA

Understanding The Basics of Radioactivity



In order to understand how radionuclides can be used in our environment, we must first understand:

1) The basics of why radioisotopes exist,
2) What is radiation and radioactive decay,
3) Key equations used to describe the radioactive decay process

Why do we care?

- Medicine
  - X Rays
  - Cancer treatments

- Industrial Applications
  - Sterilization of food, insects, etc.
  - Power

- **Marine Science Applications**
  - Age Dating
  - Proxies & Tracers

The Basics: What is in atom?

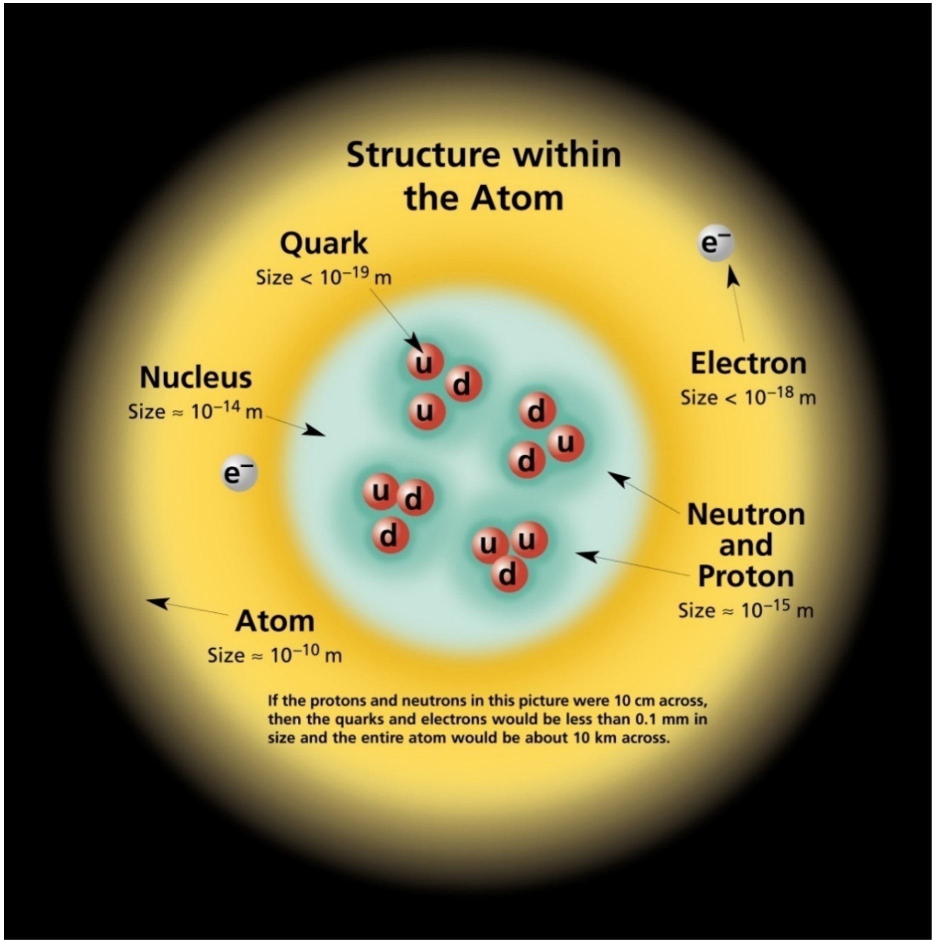
N = Neutrons, neutral charge
Z = Protons (Atomic #), positive charge
e = electrons, negative charge
A = Atomic Mass = N + Z

$\text{mass}_N = 1.008665$ amu

$\text{mass}_Z = 1.007825$ amu
$\text{mass}_e = 5.485 \times 10^{-4}$ amu

So $^{23}_{11}$Na has an A = 23, Z= 11, and N = 12

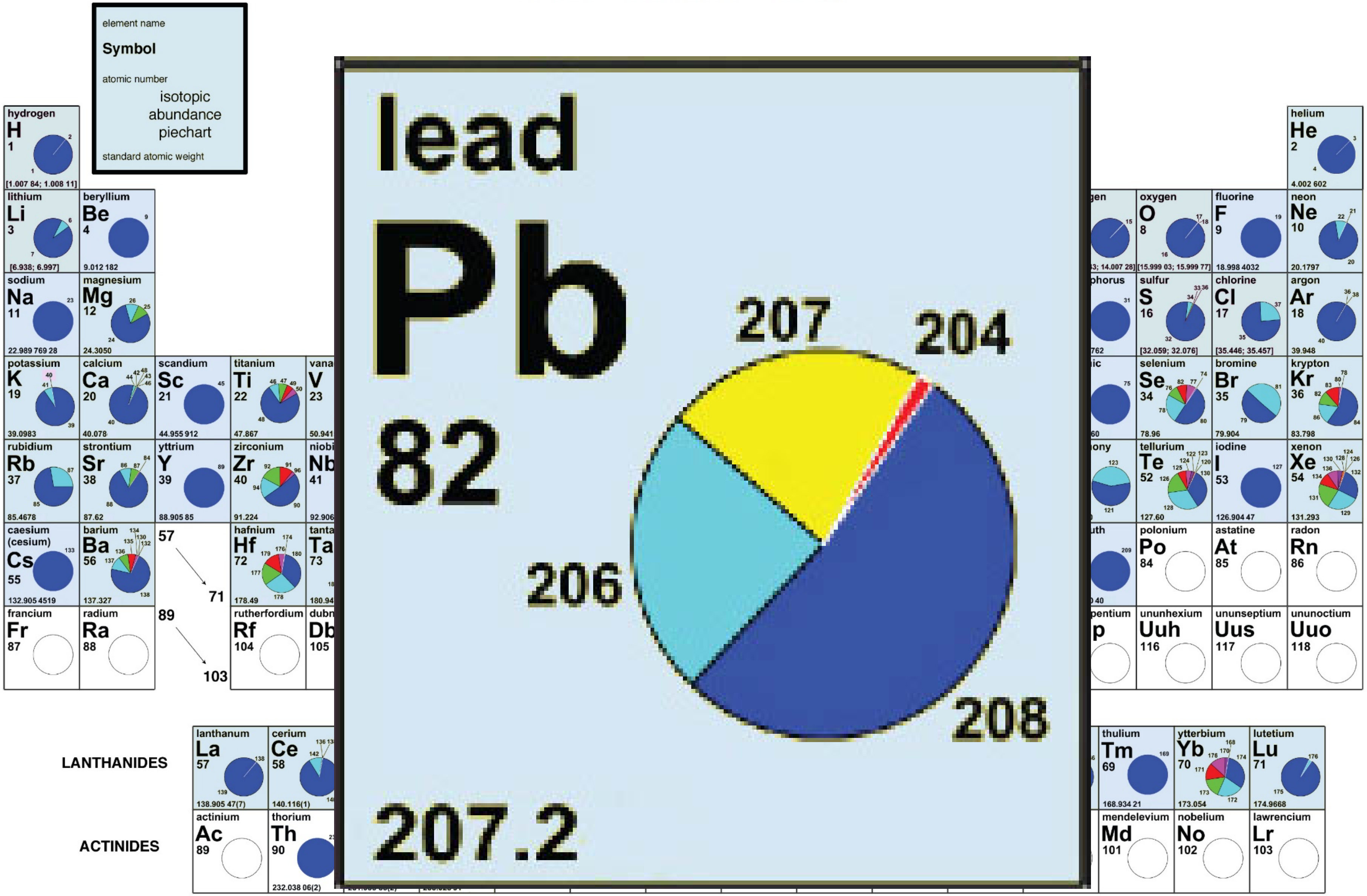
<u>Isotopes</u> have identical chemical properties but a different relative atomic mass. While the number of protons is the same, the number of neutrons in the nucleus differs.



Structure within the Atom

If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

Each pie chart shows the relative abundance of *naturally occurring* isotopes (both stable and long-lived unstable) of each element



Atomic weights of the elements 2009 (IUPAC Technical Report) Pure Appl. Chem. doi: 10.135/PAC-REP-10-09-14

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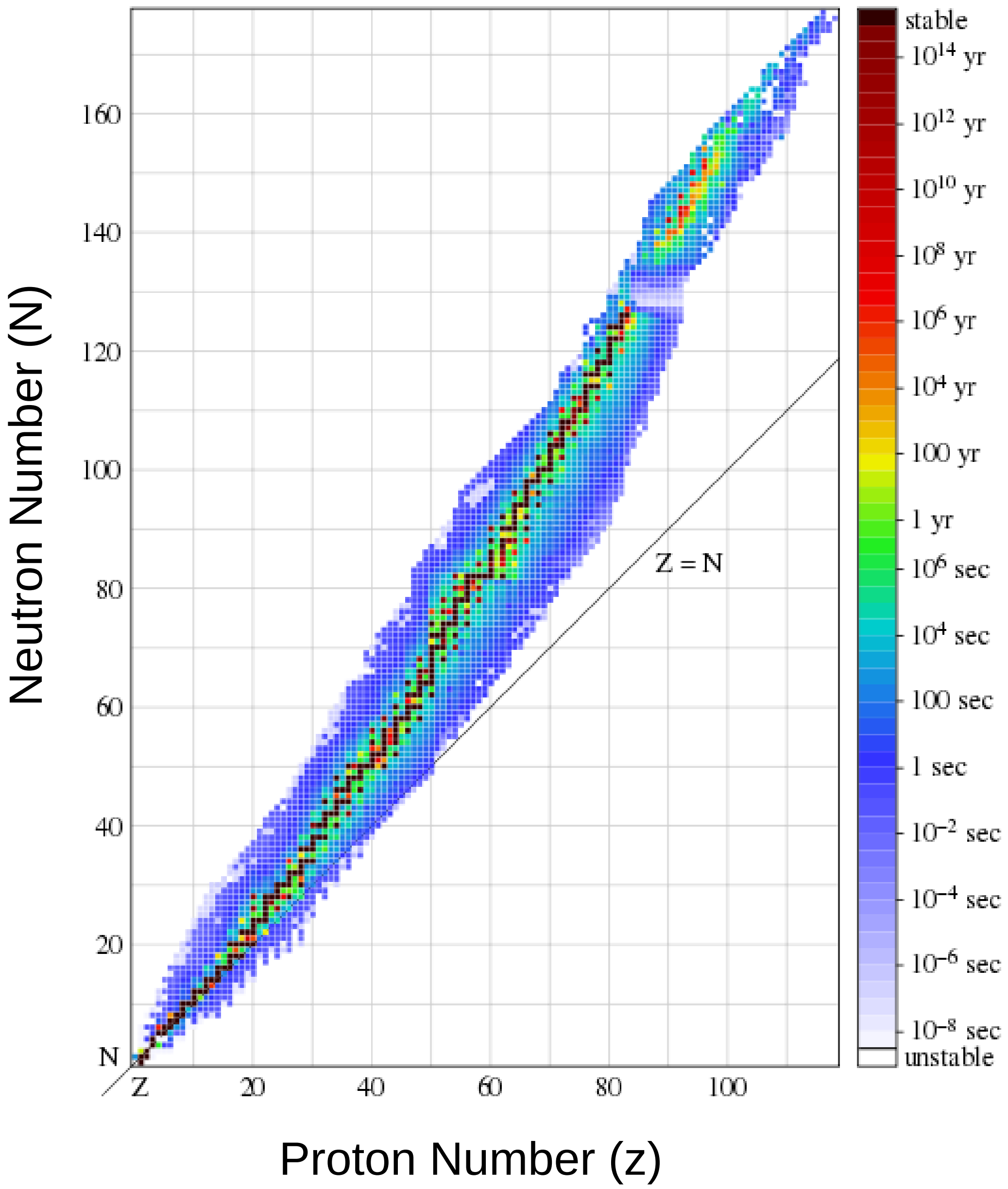
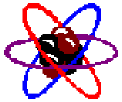
Chart of the nuclides:



Some lightweight isotopes are unstable or **radioactive** but, ***all*** elements that have an atomic number > 82 are radioactive.



Radioactivity – spontaneous change in the structure of the nucleus resulting in the transformation of the nucleus and the emission of particles (**radiation**) from the nucleus.

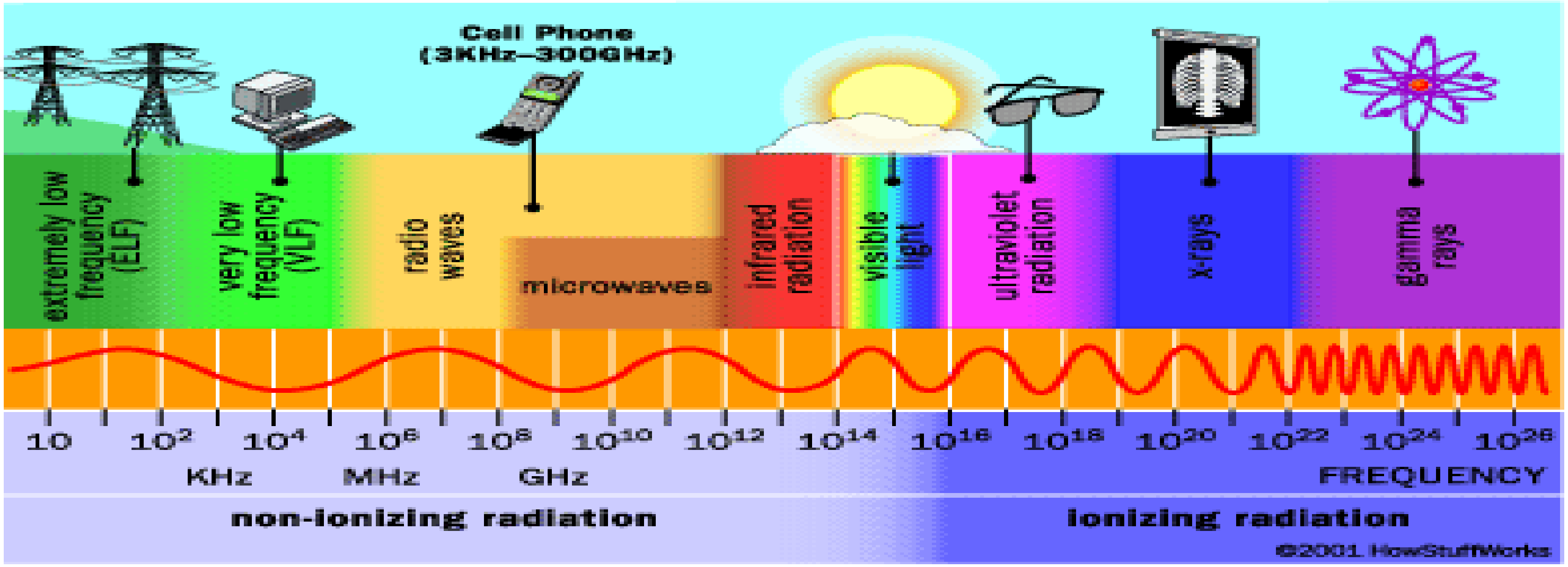
This results in a loss of energy that changes the nucleus to a more stable configuration.

There are different kinds of radiation.

Radiation is energy in the form of high speed particles (or electromagnetic waves). It can be ionizing or non-ionizing.

Low Energy

High Energy

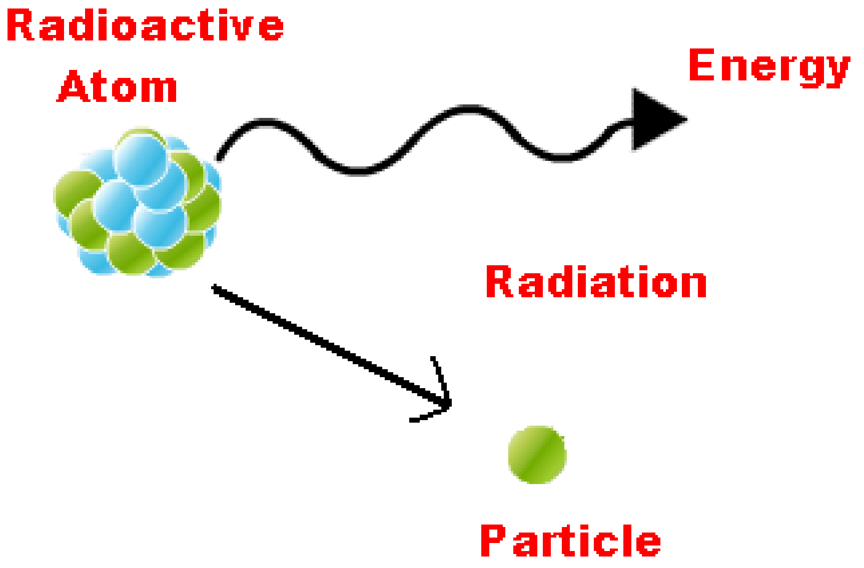


As you move from left to right, (the distance between each peak or trough) in meters **the wavelength** gets smaller. For example, the wavelength of cosmic rays is 10^{-14} meters; visible wavelengths are 10^{-6} meters; radio waves are 1 meter.

There are many different types of non-ionizing radiation, but all lack the energy to alter atoms (e.g., visible light and microwaves).

Ionizing radiation has enough energy to ionize atoms and therefore can change the normal cellular functioning.

Ionizing radiation is categorized by its strength or energy level and ***it is these particles that are emitted from an unstable or radioactive nuclide***



The Discovery of Radioactivity:
Best Failure Ever

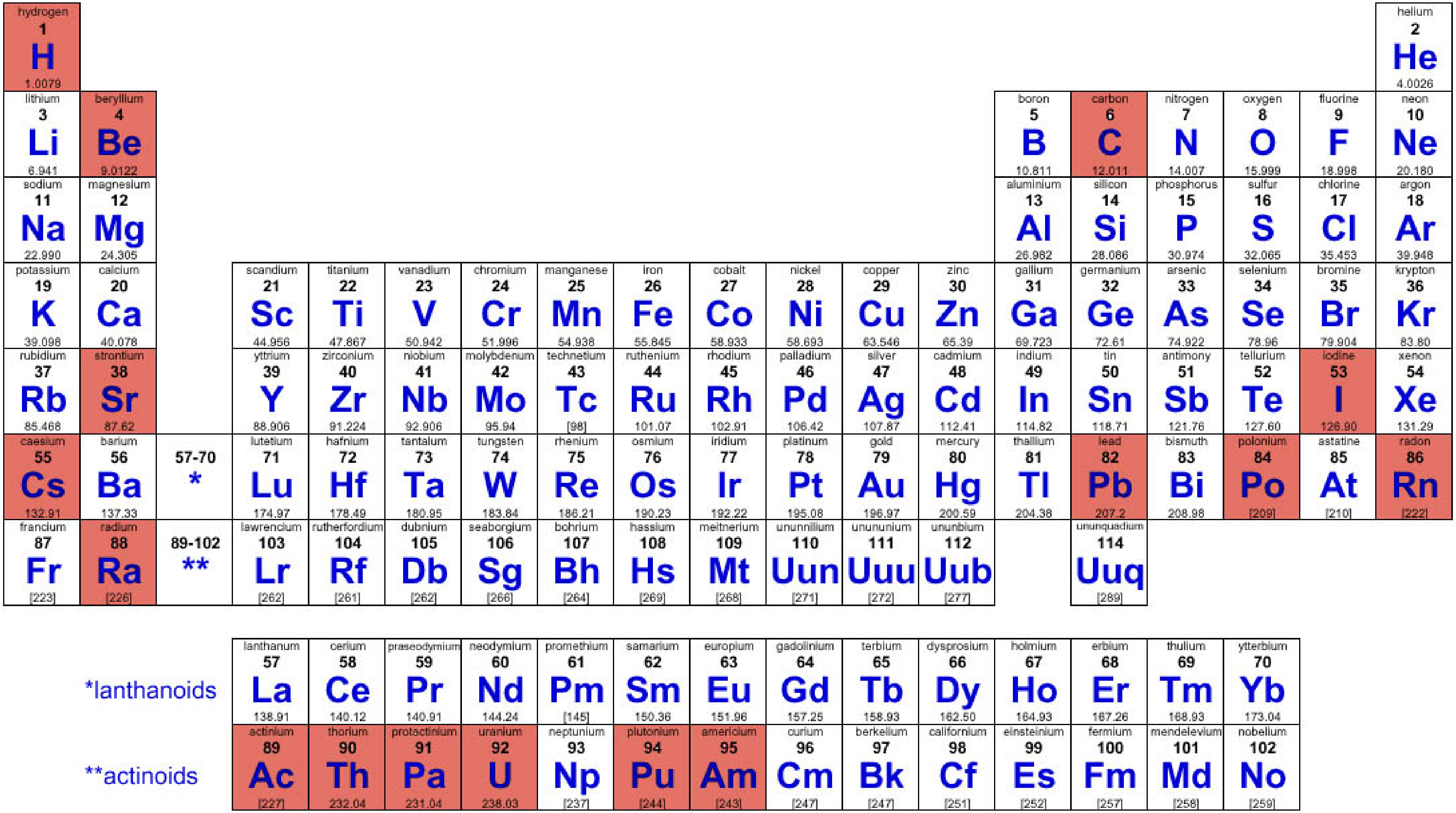
Henri Becquerel in 1896.

He exposed potassium uranyl sulfate to sunlight and then placed it on photographic plates wrapped in black paper, believing that the uranium absorbed the sun’s energy and then emitted it as x-rays.

This hypothesis was disproved on the 26th-27th of February, when his experiment "failed" because it was overcast in Paris. For some reason, Becquerel decided to develop his photographic plates anyway. To his surprise, the images were strong and clear, proving that the uranium emitted radiation without an external source of energy such as the sun. Becquerel had discovered radioactivity.

Won the Nobel Prize in Physics in 1903

So where did all of these elements come from and why do radioactive elements exist in nature?

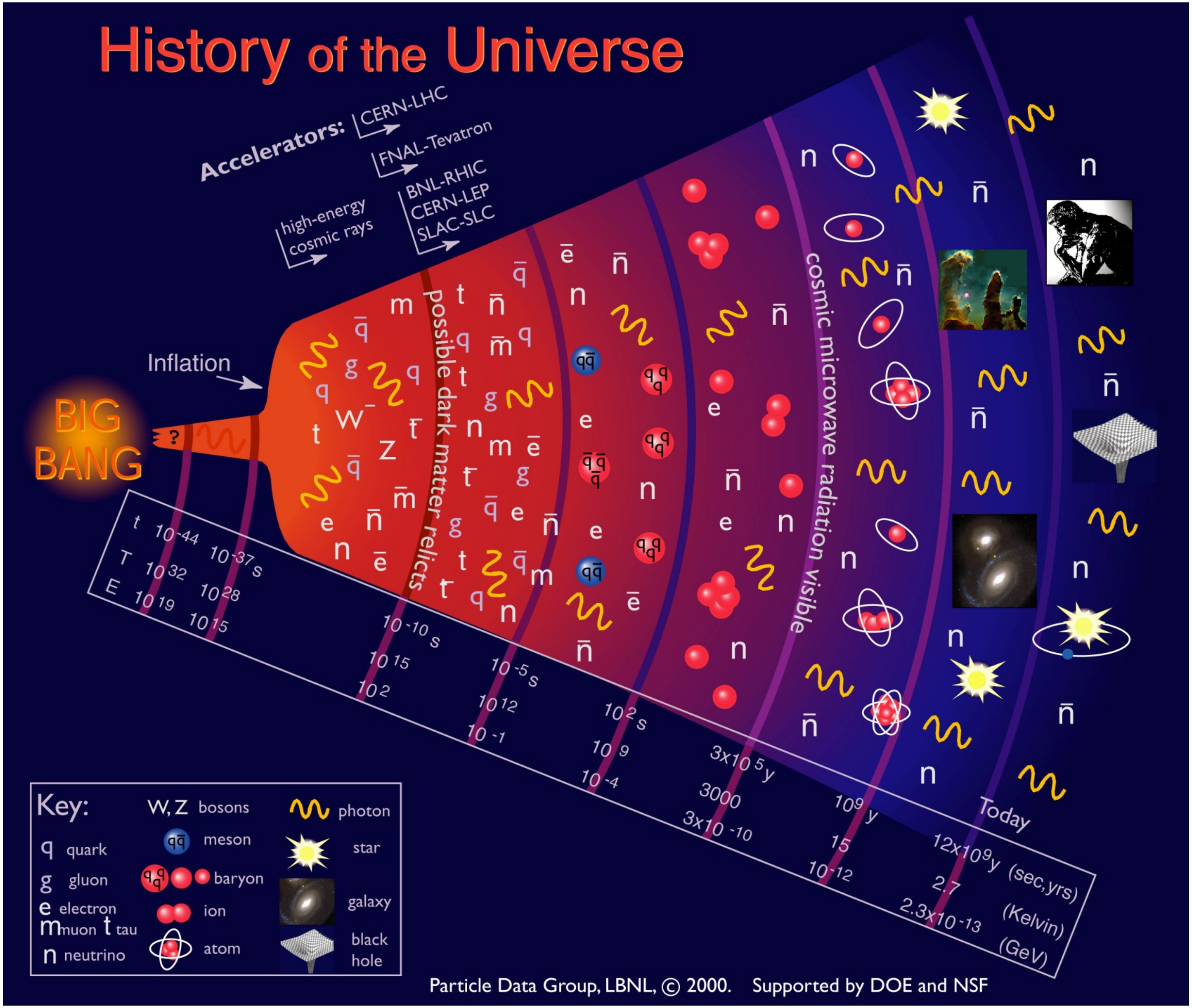


hydrogen 1 H 1.0079																		helium 2 He 4.0026
lithium 3 Li 6.941	beryllium 4 Be 9.0122												boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305												aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078		scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
rubidium 37 Rb 85.468	strontium 38 Sr 87.62		yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 *	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]
francium 87 Fr [223]	radium 88 Ra [226]	89-102 **	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	unununium 111 Uuu [272]	ununbium 112 Uub [277]		ununquadium 114 Uuq [289]				

*lanthanoids	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
**actinoids	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

The elements highlighted in **red** are the ones we are most interested in for Marine Science

Nuclear Synthesis: For our purposes a galaxy is just a large collection of gas which is gravitationally bound. This gas eventually clumps to make stars.



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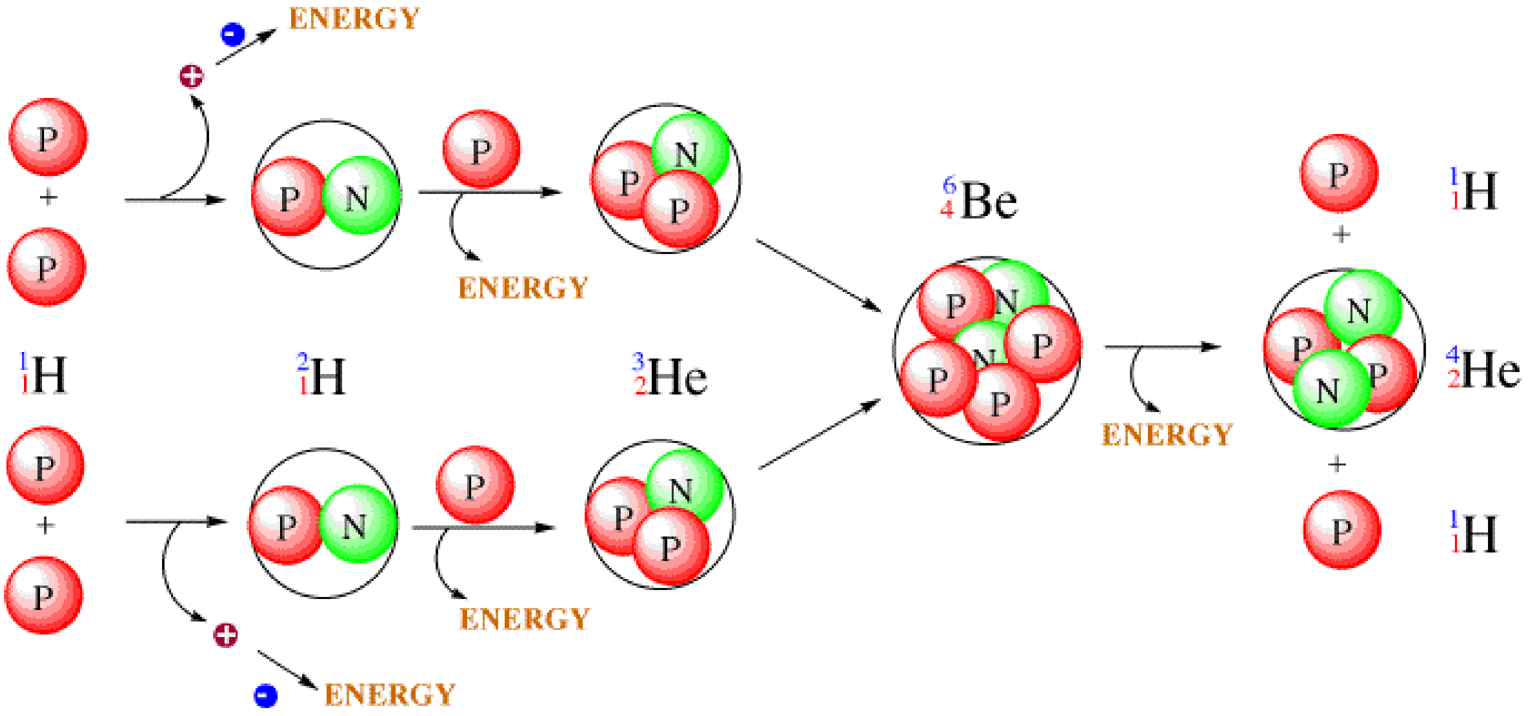
Elements are formed in 2 ways **FUSION** and **NEUTRON CAPTURE**

All stars derive their energy ***through the thermonuclear fusion of light elements in to heavy elements*****.**

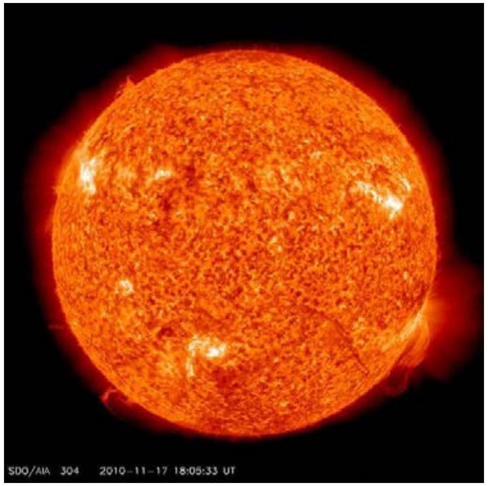
FUSION: protons, neutrons and small elements crashing together to make bigger elements (very energetic process).



This cycle yields about ~ 25 MeV (9.6 x 10^{-13} calories) of energy….

Summary of Fusion Reactions in Stellar Interiors:

Nuclear Fuel



Hydrogen Burning (> 3,000,000 K)

1) P + P → ^{2}H + positron + Energy
2) ^{2}H + P → ^{3}He + Energy
3) ^{3}He + ^{3}H → ^{4}He + P + P + Energy

Carbon Nitrogen Cycle (> 10,000,000 K)

1) ^{12}C + P → ^{13}N + Energy (1.95 MeV)
2) ^{13}N → ^{13}C + positron + Energy
3) ^{13}C + P → ^{14}N + Energy
4) ^{14}N + P → ^{15}O + Energy (7.35 MeV)
5) ^{15}O → ^{15}N + positron + Energy
6) ^{15}N + P → ^{12}C + ^{4}He + Energy (4.96 MeV)

Here P = Proton

Note that during this process, some of the neutrons that are added (which creates an isotope of the same element), converts to a proton, thus changing the element!

Oxygen Burning (> 2,000,000,000 K)

1) $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}$ + Energy
2) $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{31}\mathrm{P} + \mathrm{P}$ + Energy (7.678 MeV)
3) $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{31}\mathrm{S} + {}^{4}\mathrm{He}$ + Energy (1.500 MeV)
4) $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He}$ + Energy (9.594 MeV)

Silicon Burning (> 3,000,000,000 K)

1) $^{28}\mathrm{Si} + {}^{28}\mathrm{Si} \rightarrow 7\,({}^{4}\mathrm{He})$ + Energy
2) $^{28}\mathrm{Si} + 7\,({}^{4}\mathrm{He}) \rightarrow {}^{56}\mathrm{Ni}$ + Energy
3) $^{28}\mathrm{Si} + {}^{28}\mathrm{Si} \rightarrow {}^{56}\mathrm{Ni}$+ Energy
4) $^{56}\mathrm{Ni} \rightarrow {}^{56}\mathrm{Co}$ + positron + Energy
5) $^{56}\mathrm{Co} \rightarrow {}^{56}\mathrm{Fe}$ + positron + Energy

$^{56}\mathrm{Fe}$ = 26 Protons + 30 Neutrons
$^{56}\mathrm{Co}$ = 27 P + 29 N
$^{56}\mathrm{Ni}$ = 28 P + 28 N

After Fe, fusion becomes increasingly difficult…

Neutron Capture (less energetic): Two processes:

R-process --> Rapid = capture of a neutron before a neutron to proton decay can occur (neutron $t_{1/2}$ = 12 -15 minutes!!)

S-process --> Slow = Neutron capture --> decays into proton --> another neutron is captured

1) $^{56}\text{Fe} + \text{N} \rightarrow {}^{57}\text{Fe}$ + Energy
2) $^{57}\text{Fe} + \text{N} \rightarrow {}^{58}\text{Fe}$ + Energy
3) $^{58}\text{Fe} + \text{N} \rightarrow {}^{59}\text{Fe}$ + Energy

S-process

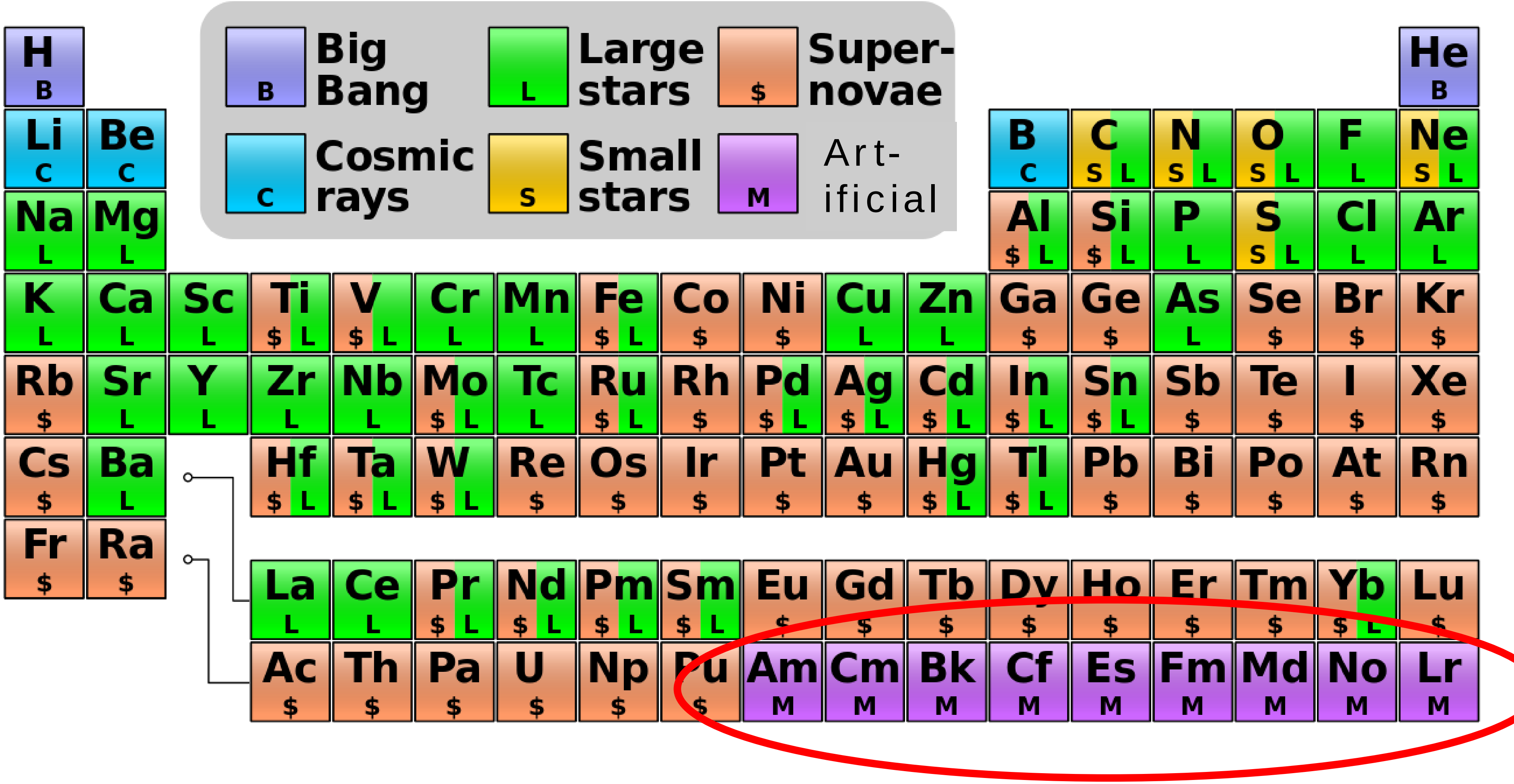
$^{59}\text{Fe} \rightarrow {}^{59}\text{Co}$ + electron + Energy

R-process

$^{59}\text{Fe} + \text{N} \rightarrow {}^{60}\text{Fe} + \text{N} \rightarrow {}^{61}\text{Fe}$

$^{61}\text{Fe} \rightarrow {}^{61}\text{Co}$ + electron + Energy

Put it all together….



B Big Bang
L Large stars
$ Supernovae
C Cosmic rays
S Small stars
M Artificial

H B																	He B
Li C	Be C											B C	C S L	N S L	O S L	F L	Ne S L
Na L	Mg L											Al $ L	Si $ L	P L	S S L	Cl L	Ar L
K L	Ca L	Sc L	Ti $ L	V $ L	Cr L	Mn L	Fe $ L	Co $	Ni $	Cu L	Zn L	Ga $	Ge $	As L	Se $	Br $	Kr $
Rb $	Sr L	Y L	Zr L	Nb L	Mo $ L	Tc L	Ru $ L	Rh $	Pd $ L	Ag $ L	Cd $ L	In $ L	Sn $ L	Sb $	Te $	I $	Xe $
Cs $	Ba L		Hf $ L	Ta $ L	W $ L	Re $	Os $	Ir $	Pt $	Au $	Hg $ L	Tl $ L	Pb $	Bi $	Po $	At $	Rn $
Fr $	Ra $																
			La L	Ce L	Pr $ L	Nd $ L	Pm $ L	Sm $ L	Eu $	Gd $	Tb $	Dy $	Ho $	Er $	Tm $	Yb $ L	Lu $
			Ac $	Th $	Pa $	U $	Np $	Pu $	Am M	Cm M	Bk M	Cf M	Es M	Fm M	Md M	No M	Lr M

These were generated during Supernovae and now only produced artificially

What are the characteristics of a nucleus that determines stability? Four main processes….

1) Spin Pairing (+)

Neutrons and protons are *fermions.* They have a spin (± ½) and they like to form pairs (Pauli Exclusion Principle)

A (n+z)	Z (# protons)	N (# neutrons)	# of Stable Isotopes
Even	Even	Even	156
Odd	Even	Odd	50
Odd	Odd	Even	48
Even	Odd	Odd	5

H, Li, B, N, Ta

2) Shell Binding (+)

Orbitals (electronic and nuclear) like to be filled! “Magic Numbers” are when those orbitals are completely filled.

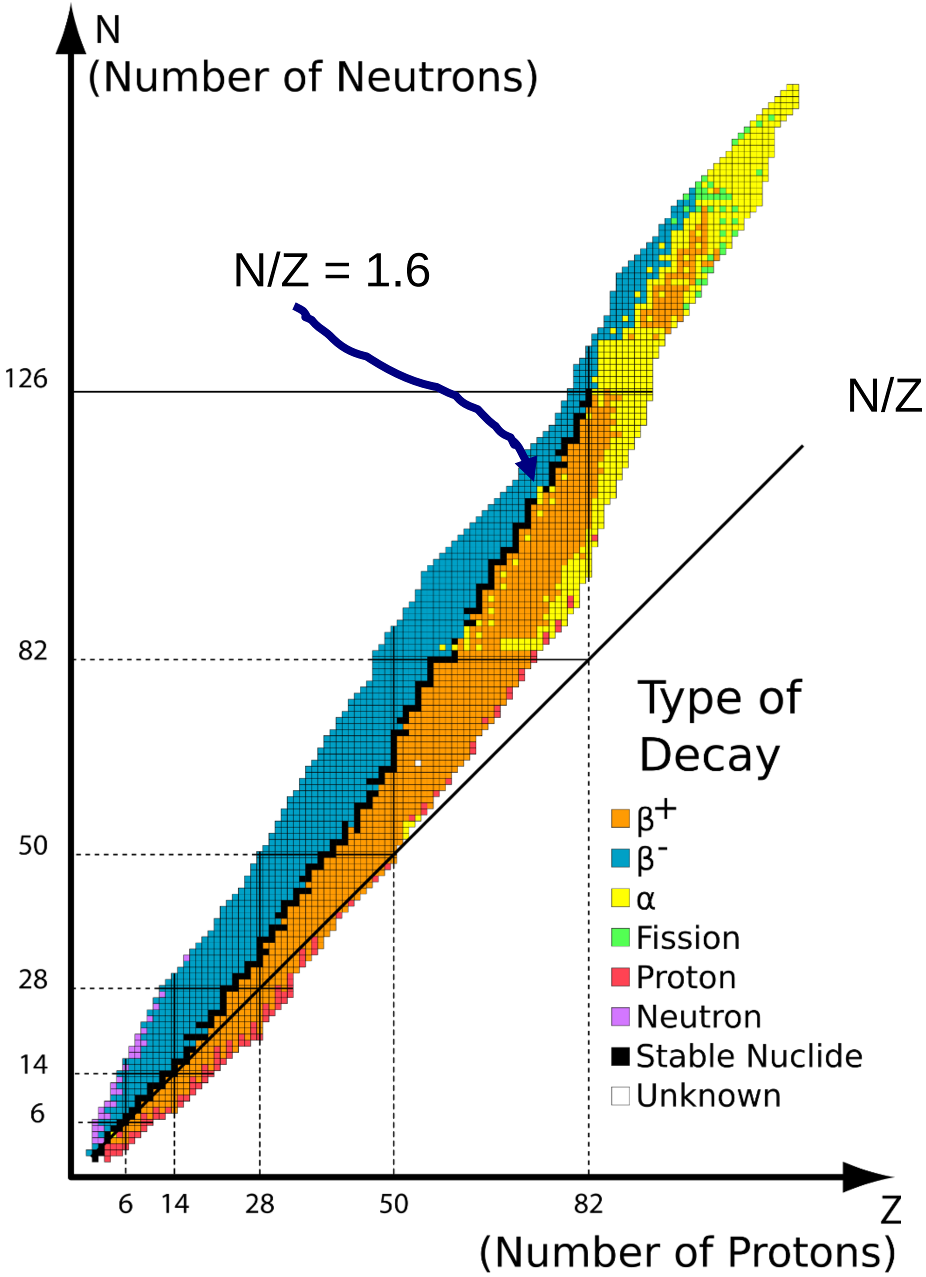
2 (S), **8** (S+P), **20** (S+P+D), **28**, **50**, **82**, **126**

For example

^{19}K has 3 stable isotopes
^{20}Ca has 6 stable isotopes **MAGIC!**
^{21}Sc has 1 stable isotope

3) Surface Tension (-)

Surface tension is related to size. The higher the number of neutrons and protons, the lower the surface tension.



4) Coulomb Repulsion (-)

Like charges repel, meaning that a nucleus with more protons has more internal repulsion. As a result, it is easier to add neutrons (no charge) versus protons which are + charged!

Binding Energy reflects these four processes and is the energy that would be required to disassemble the nucleus of an atom into its component parts (be it protons, atoms, etc).

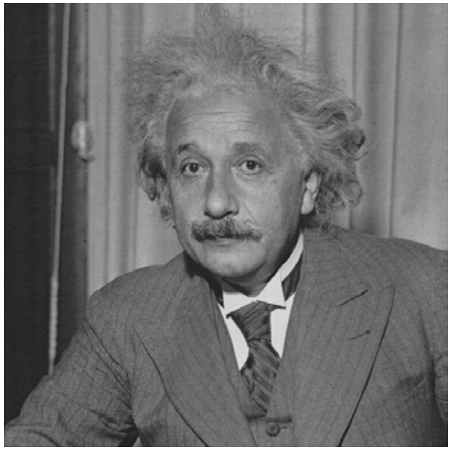
Ever notice that when you add the number of protons and neutrons together in an atom and compare it to what it actually weighs, you get MORE?

This is called the **mass defect = ΔM.**

EXAMPLE:

${}^{23}_{11}$Na = (11 x 1.007825) + (12 x 1.008665) = 23.19006 amu

Actual ${}^{23}_{11}$Na = 22.98977 amu **Δ M = 0.20236 amu**

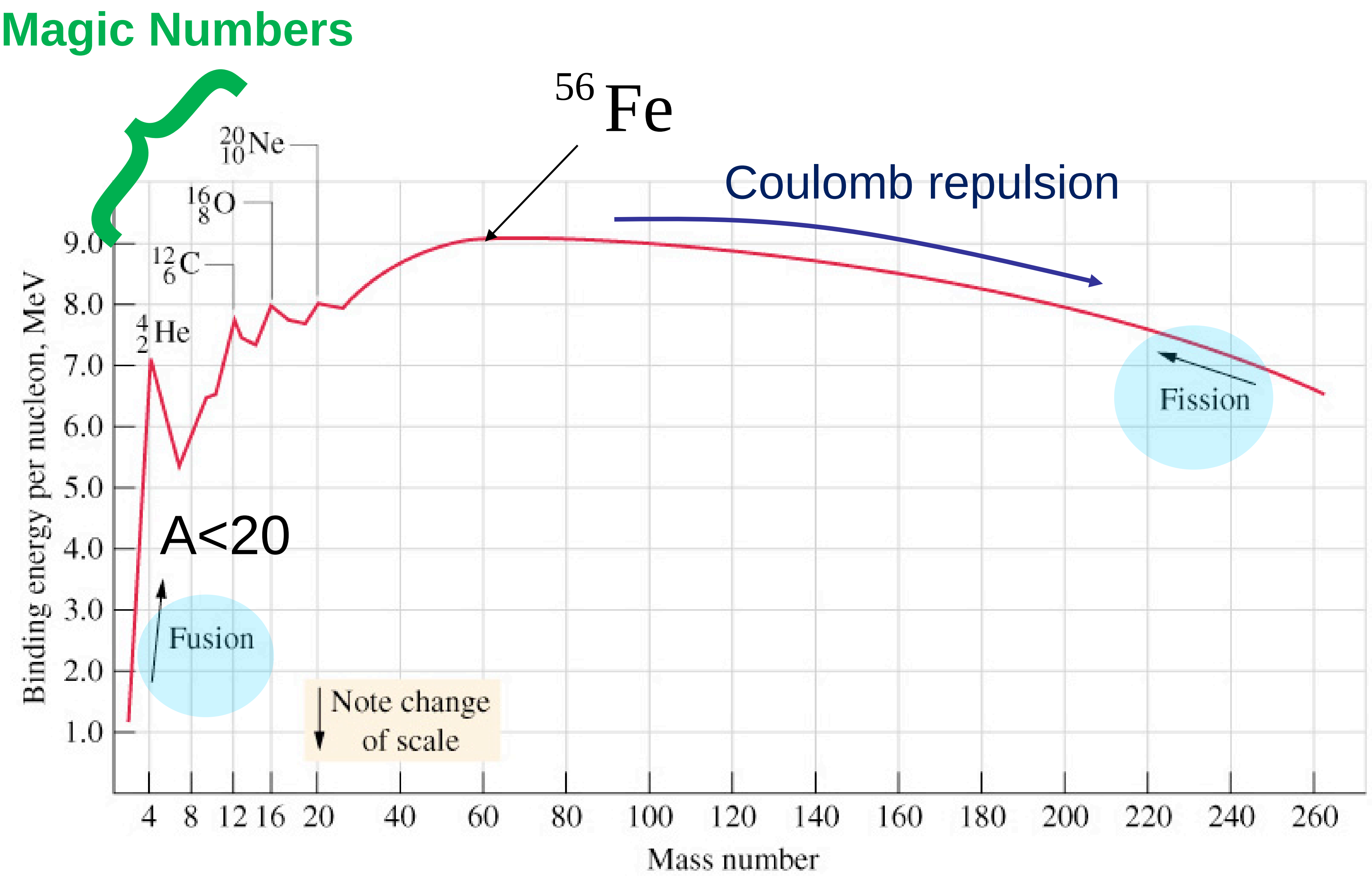


Theory of Relativity!!

Energy Released = ΔMc^2

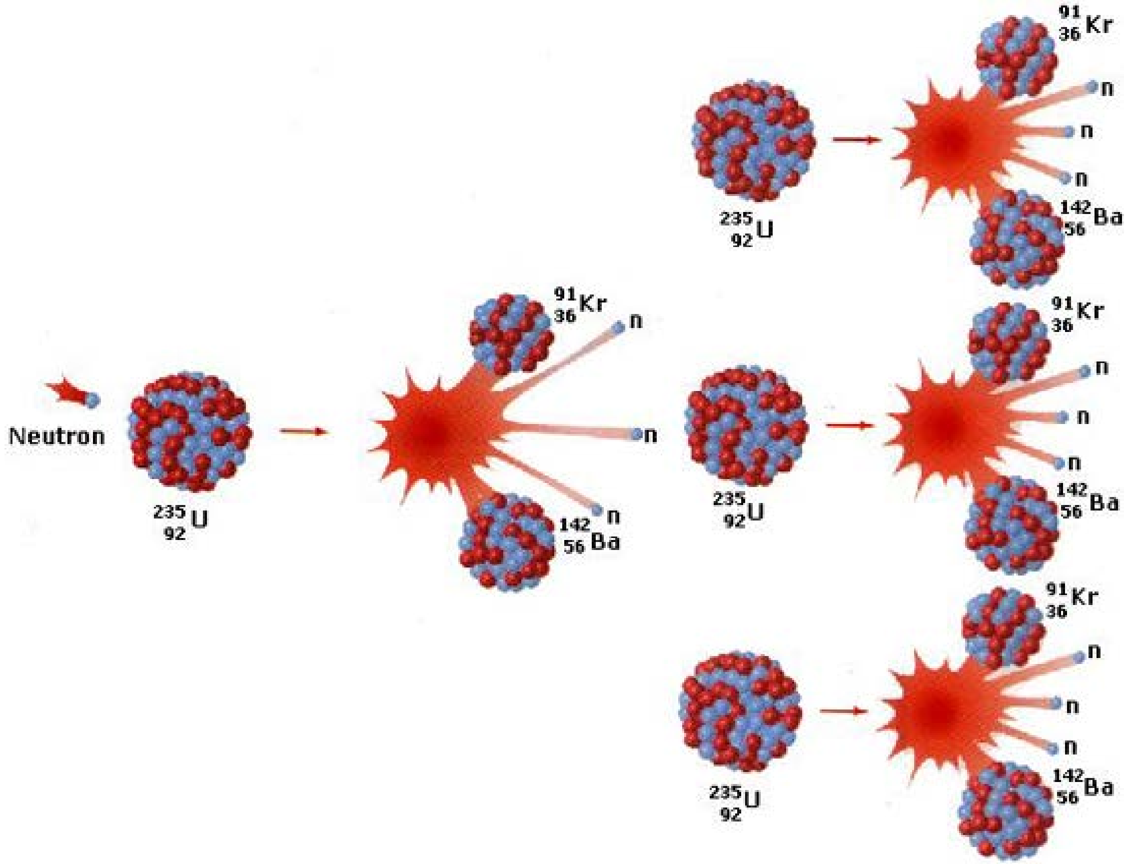
Where **c** is the speed of the light

Curve of the Binding Energy per nucleon



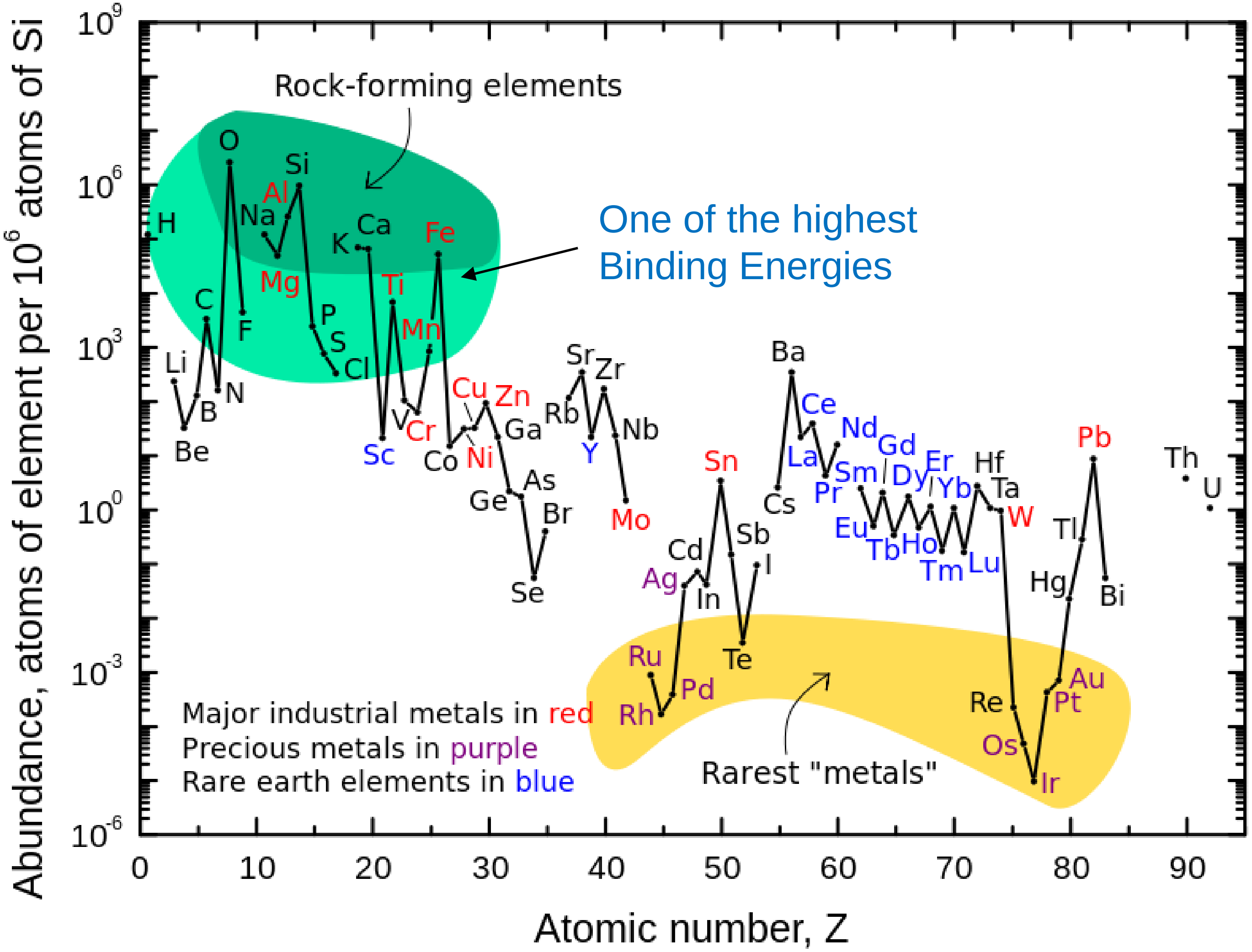
One last process….**Fission**.

Either a nuclear reaction or a ***radioactive decay process*** in which the nucleus of an atom splits into smaller parts (lighter nuclei). The fission process often produces free neutrons and photons and releases a very large amount of energy even by the energetic standards of radioactive decay.



Element Abundance therefore depends on a mixture of Binding Energy and formation mechanisms.

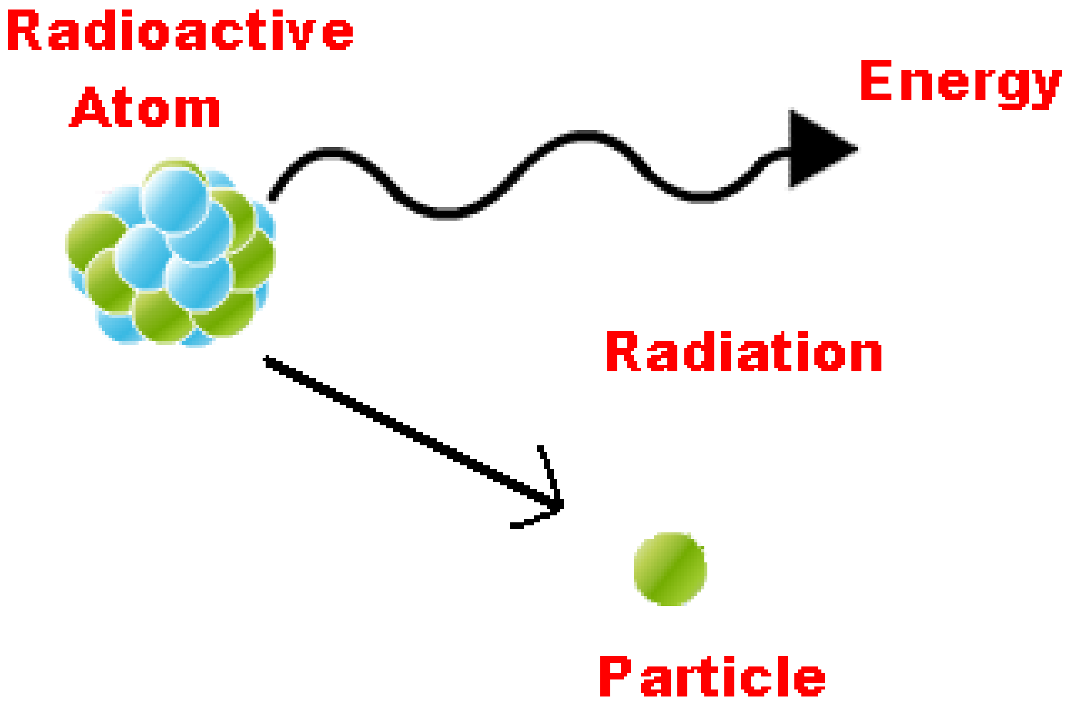
- abundances of first 50 elements decrease exponentially with atomic #
- abundances of the heavier elements *independent* of atomic number
- note anomalously high abundance of Fe



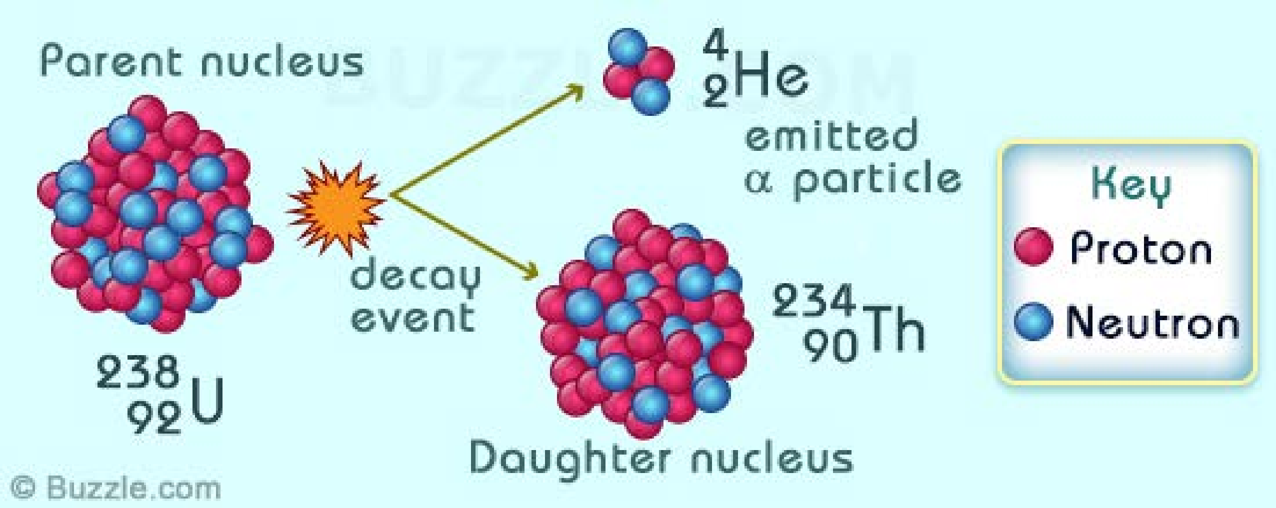
One of the highest Binding Energies

So now that we know HOW radioactive elements formed and why they exist in nature, let's examine the specific decay mechanisms that remove their excess energy to become more stable.

Nuclei will transform in such a way to increase its Binding Energy per nucleon!



Alpha Decay



Emission of a helium nucleus, which contains two protons and two neutrons (but no electrons).

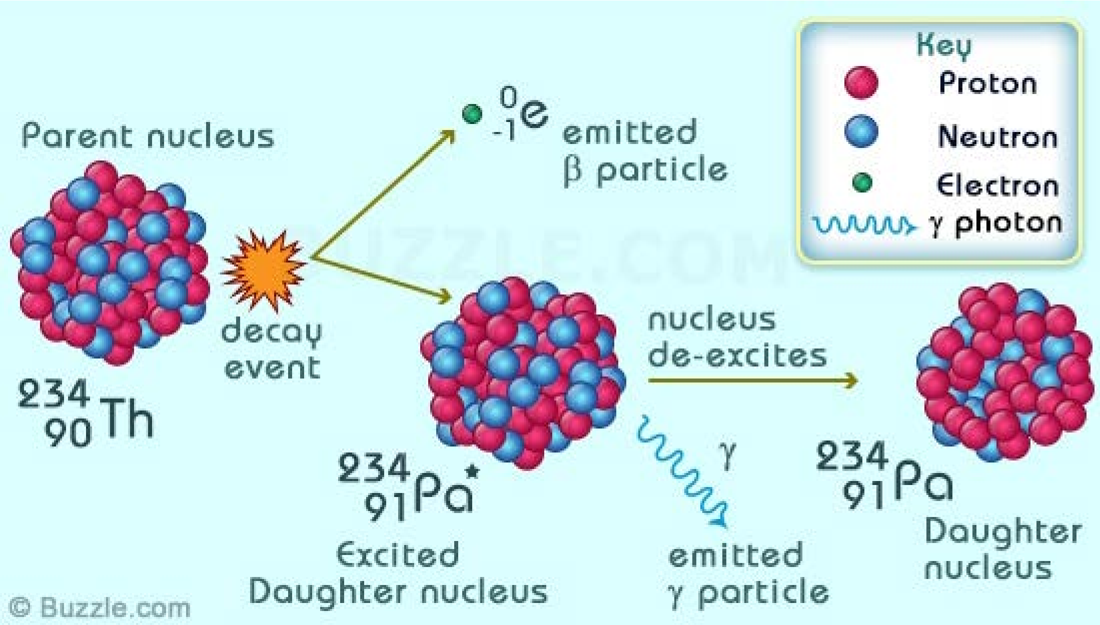
$$^{238}_{92}\mathrm{U} \text{ --> } {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} + \gamma + \text{Energy}$$

*note the gamma

α-particle takes most (but not all) of the decay energy (it is the lightest)

Alpha decay occurs predominantly with A > 82 and is specific to the radionuclude

Beta Decay



1) $^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta^{-} + \nu + \gamma +$ *Energy* (Negatron)

neutrino

Conversion of a neutron into a proton and a beta particle escapes (a high-energy electron) from the nucleus. Note that the *mass* number *does not change* and there is a negligible effect on atomic weight.

2) $^{40}_{19}\mathrm{K} + \beta^{-} \rightarrow {}^{40}_{18}\mathrm{Ar} +$ *Energy* (Electron Capture)

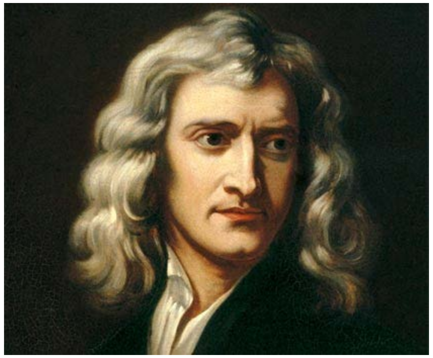
Only occurs predominantly with nuclei that have excess neutrons above a stable conformation

Beta particles are emitted with a range of energies

Remember the Conservation of Momentum Law???

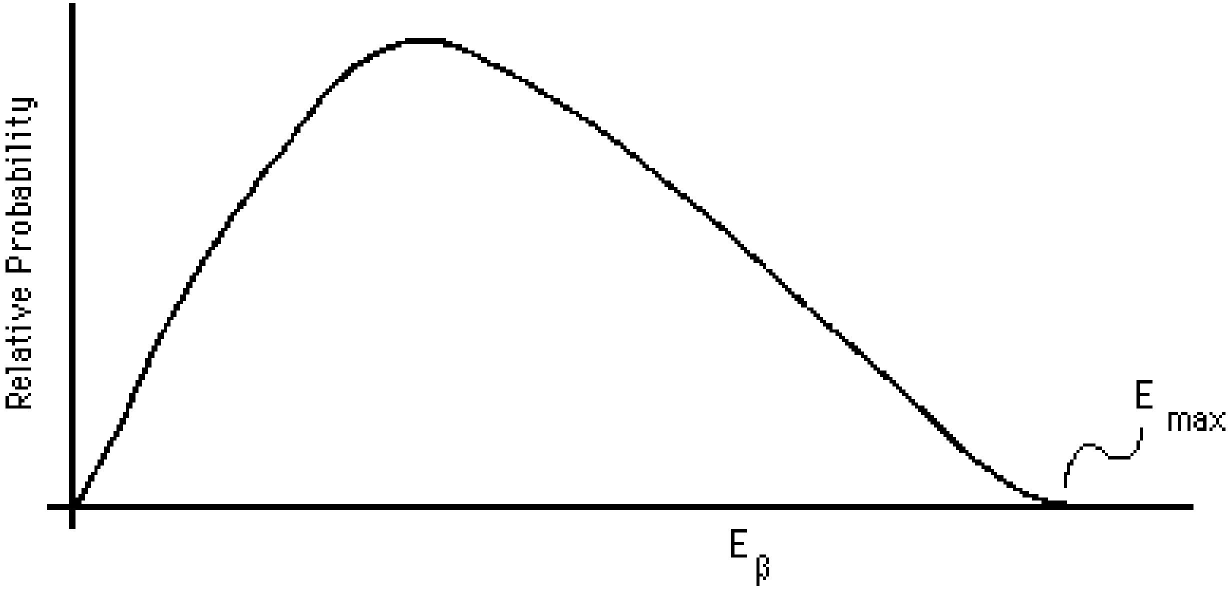
Now have *three* particles to deal with:

$${}_{z}P_{arent} \rightarrow {}_{z+1}D_{aughter} + \beta + \nu$$

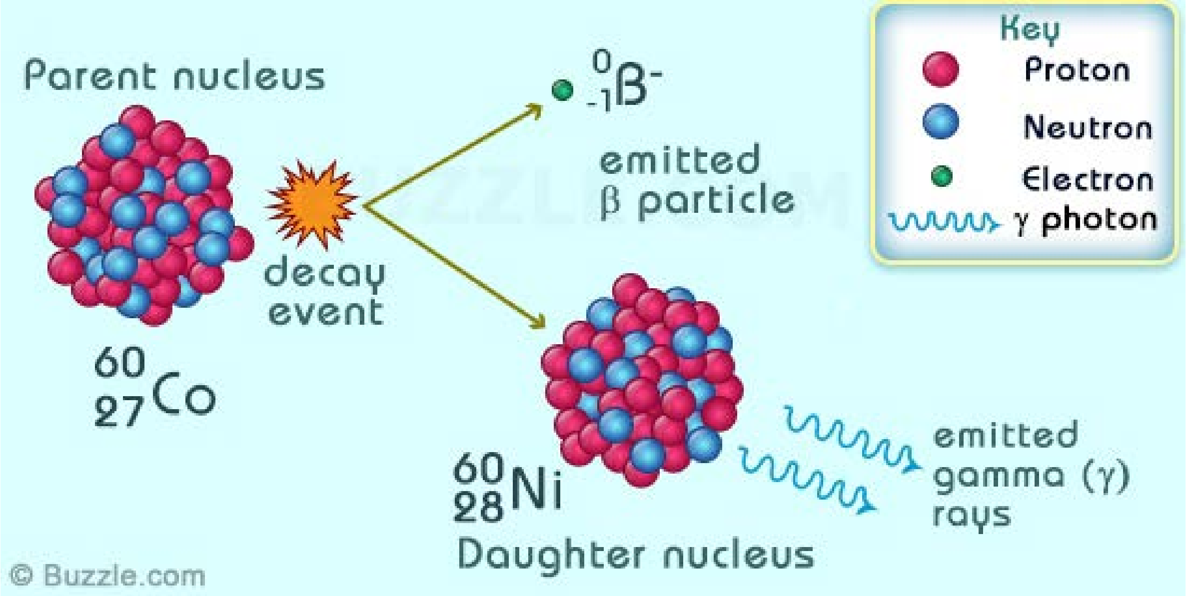


Isaac Newton

This results in an *infinite* number of ways to share momentum



Gamma Decay



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$$^{7}_{4}Be' \rightarrow \; ^{7}_{4}Be + \gamma + \mathit{Energy}$$

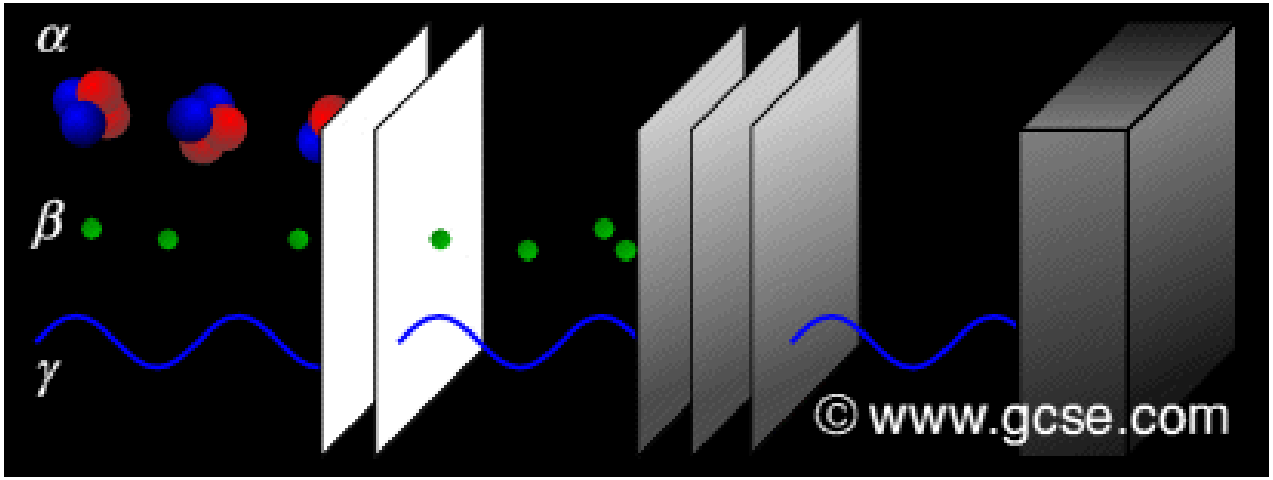
Conversion of nuclear energy to electromagnetic energy (and the loss of energy (photon) from a neutron), note that the atomic number *does not change*.

Nearly always occurs with alpha and beta emissions and energy is specific to the radionuclide

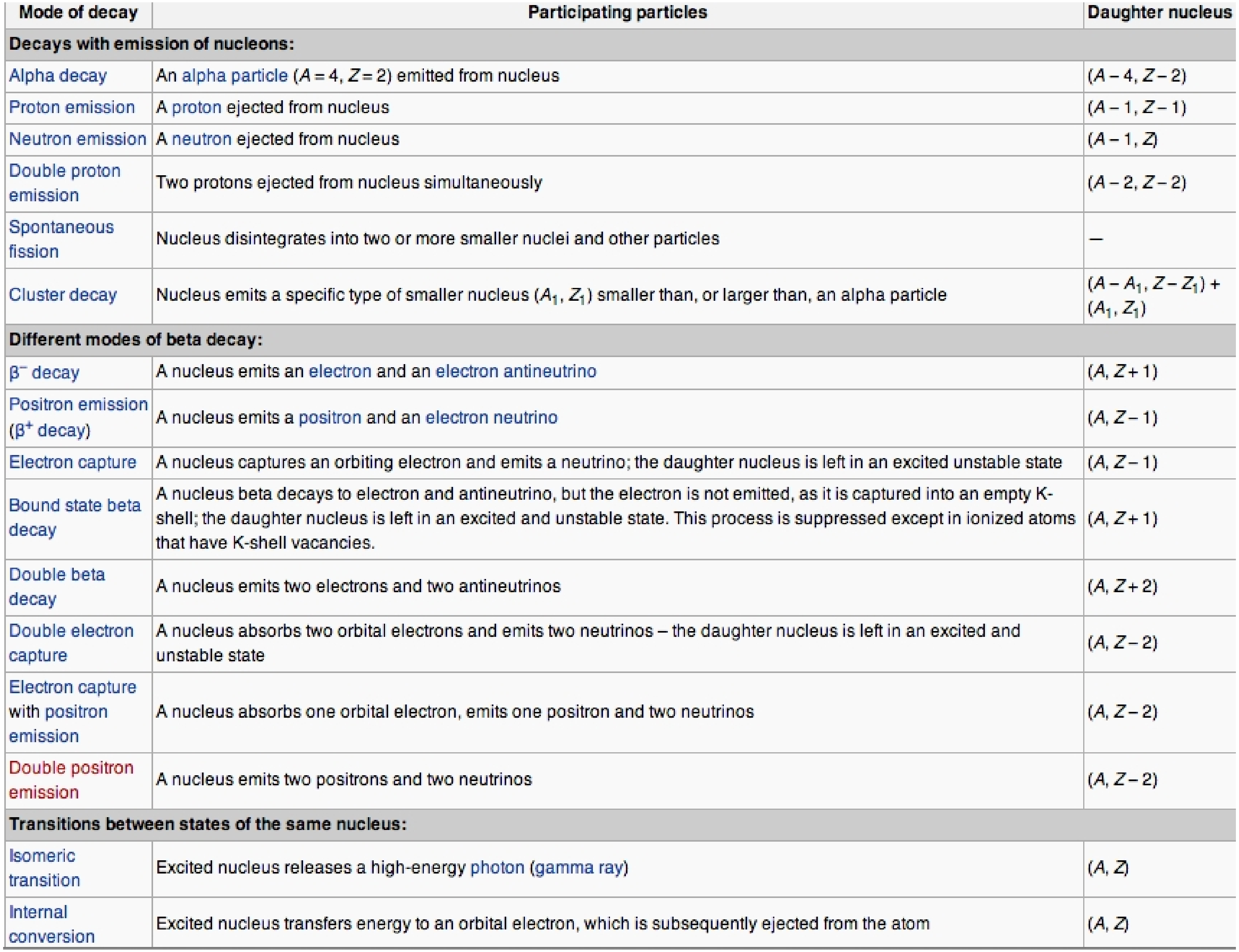
Alpha (α) particles Most densely ionizing, but least penetrating. This means that cells can be protected or shielded from damage by alpha particles by clothing. Even the dead outer layer of your skin will protect you from damage from alpha particles. However, if alpha emitters are inhaled or ingested or get into a cut on the skin, they can cause damage to cells. As alpha particles are emitted inside the body, the surrounding cells are damaged.

Beta (β) particles More energetic. Can travel several feet through air, but are stopped with denser materials such as wood, glass or aluminum foil depending on their energy. They can travel a few mm inside tissue.

Gamma (γ) rays High-energy electromagnetic energy waves and the most penetrating type of radiation. Cells must be shielded from gamma rays with concrete, lead or steel. Not all may do cellular damage, but they must interact with the material to do so.



There are many different types of particles that are emitted from the nucleus during radioactive decay.



Mode of decay	Participating particles	Daughter nucleus
Decays with emission of nucleons:		
Alpha decay	An alpha particle ($A = 4$, $Z = 2$) emitted from nucleus	($A - 4$, $Z - 2$)
Proton emission	A proton ejected from nucleus	($A - 1$, $Z - 1$)
Neutron emission	A neutron ejected from nucleus	($A - 1$, Z)
Double proton emission	Two protons ejected from nucleus simultaneously	($A - 2$, $Z - 2$)
Spontaneous fission	Nucleus disintegrates into two or more smaller nuclei and other particles	—
Cluster decay	Nucleus emits a specific type of smaller nucleus (A_1, Z_1) smaller than, or larger than, an alpha particle	($A - A_1$, $Z - Z_1$) + (A_1, Z_1)
Different modes of beta decay:		
β^- decay	A nucleus emits an electron and an electron antineutrino	(A, $Z + 1$)
Positron emission (β^+ decay)	A nucleus emits a positron and an electron neutrino	(A, $Z - 1$)
Electron capture	A nucleus captures an orbiting electron and emits a neutrino; the daughter nucleus is left in an excited unstable state	(A, $Z - 1$)
Bound state beta decay	A nucleus beta decays to electron and antineutrino, but the electron is not emitted, as it is captured into an empty K-shell; the daughter nucleus is left in an excited and unstable state. This process is suppressed except in ionized atoms that have K-shell vacancies.	(A, $Z + 1$)
Double beta decay	A nucleus emits two electrons and two antineutrinos	(A, $Z + 2$)
Double electron capture	A nucleus absorbs two orbital electrons and emits two neutrinos – the daughter nucleus is left in an excited and unstable state	(A, $Z - 2$)
Electron capture with positron emission	A nucleus absorbs one orbital electron, emits one positron and two neutrinos	(A, $Z - 2$)
Double positron emission	A nucleus emits two positrons and two neutrinos	(A, $Z - 2$)
Transitions between states of the same nucleus:		
Isomeric transition	Excited nucleus releases a high-energy photon (gamma ray)	(A, Z)
Internal conversion	Excited nucleus transfers energy to an orbital electron, which is subsequently ejected from the atom	(A, Z)

A single radioisotope can decay by many pathways….. but they are **SET** pathways!

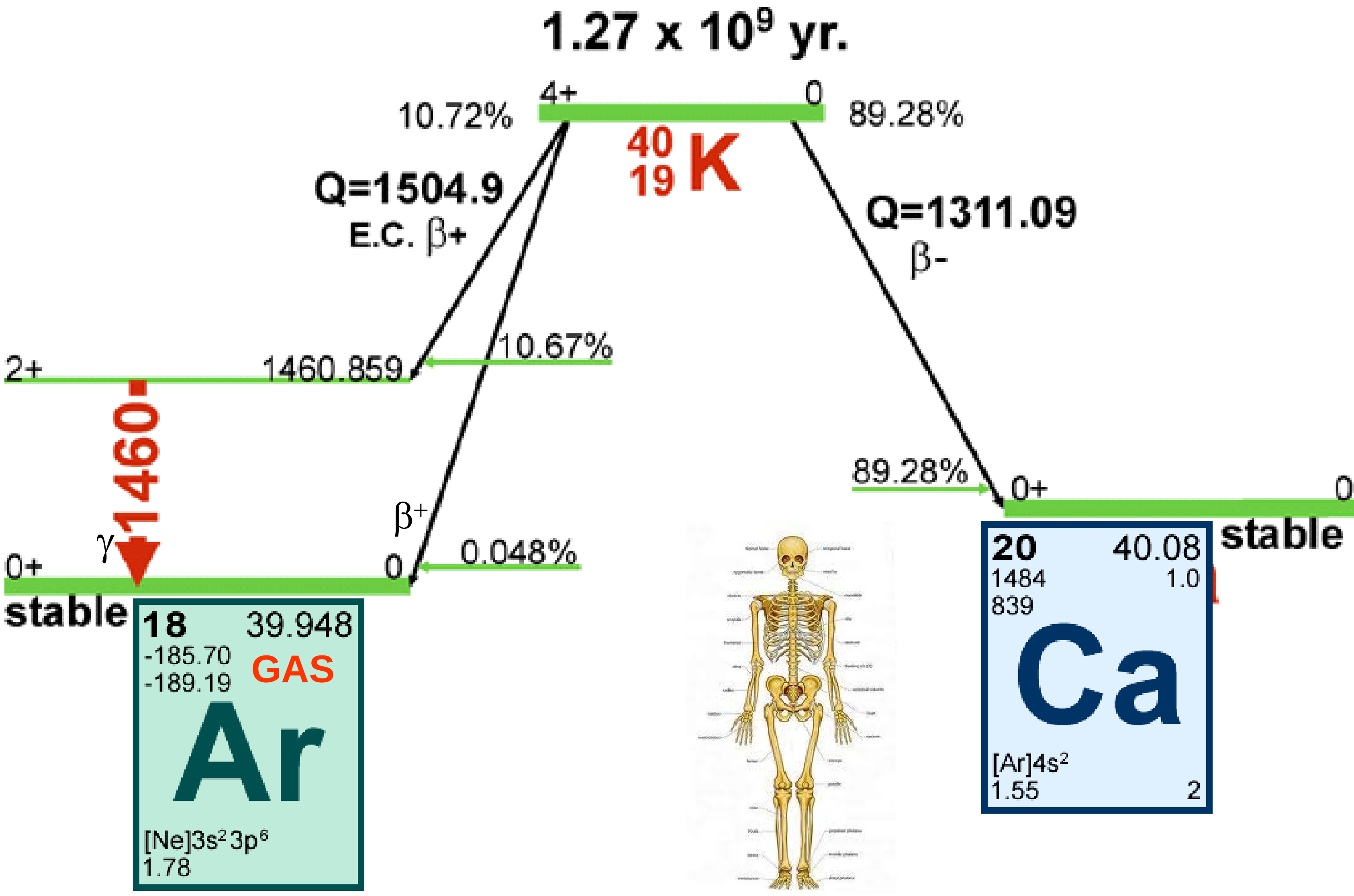
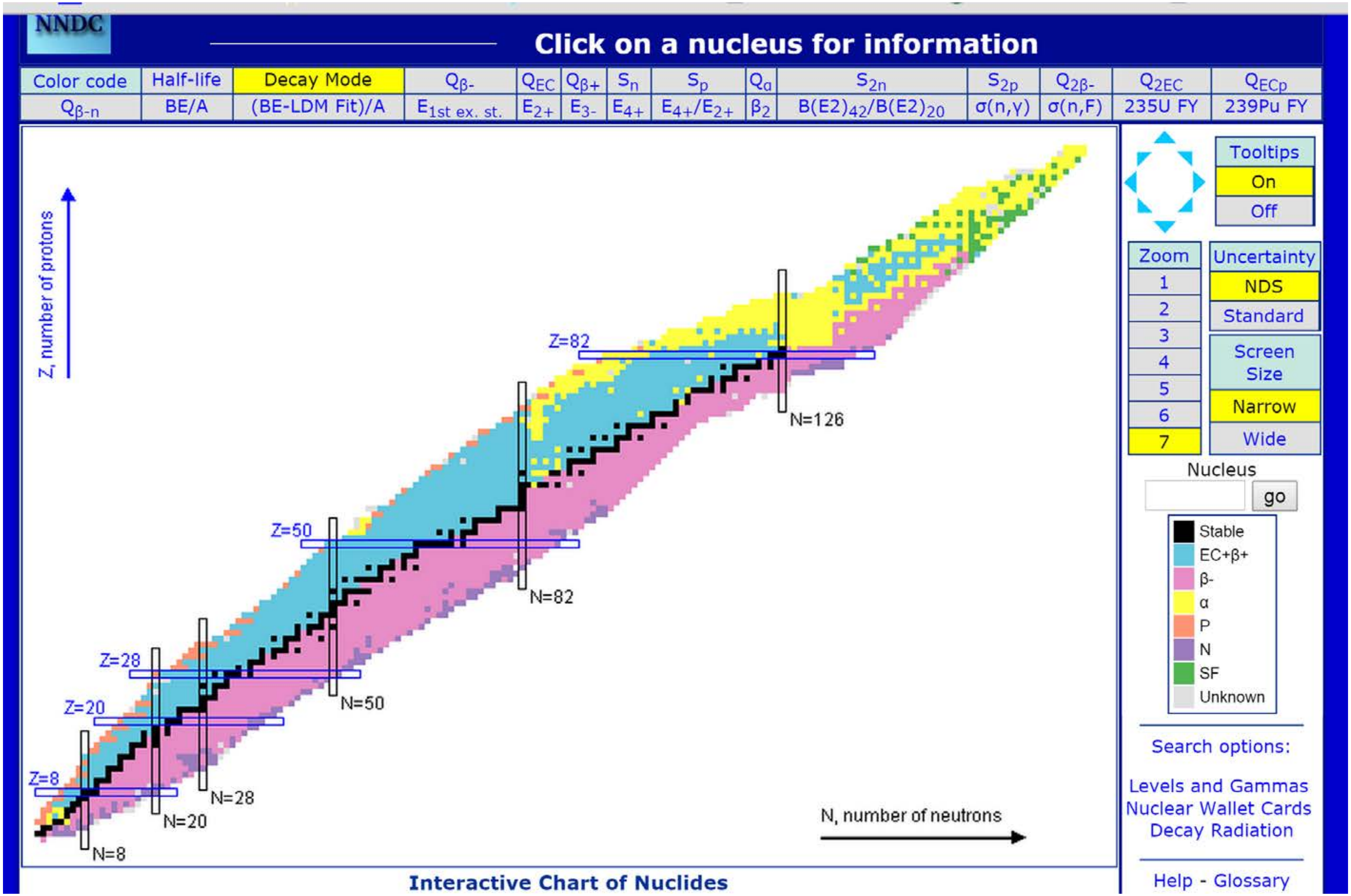
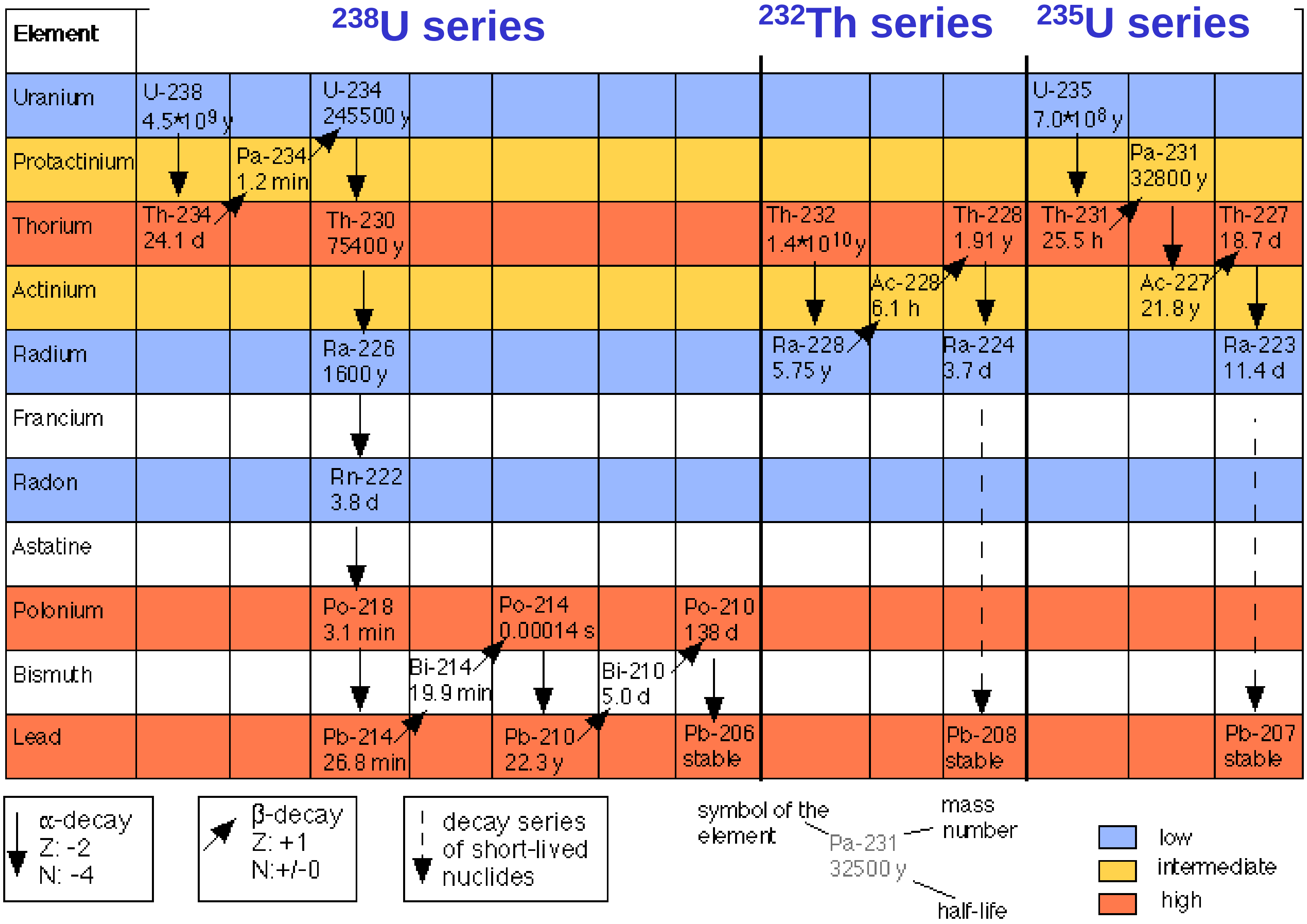


Chart of the Nuclides



http://www.nndc.bnl.gov/chart

U-Th series decay chains



Element	^{238}U series							^{232}Th series			^{235}U series		
Uranium	U-238 4.5*10^9 y		U-234 245500 y								U-235 7.0*10^8 y		
Protactinium		Pa-234 1.2 min										Pa-231 32800 y	
Thorium	Th-234 24.1 d		Th-230 75400 y					Th-232 1.4*10^{10} y		Th-228 1.91 y	Th-231 25.5 h		Th-227 18.7 d
Actinium									Ac-228 6.1 h			Ac-227 21.8 y	
Radium			Ra-226 1600 y					Ra-228 5.75 y		Ra-224 3.7 d			Ra-223 11.4 d
Francium													
Radon			Rn-222 3.8 d										
Astatine													
Polonium			Po-218 3.1 min		Po-214 0.00014 s		Po-210 138 d						
Bismuth				Bi-214 19.9 min		Bi-210 5.0 d							
Lead			Pb-214 26.8 min		Pb-210 22.3 y		Pb-206 stable			Pb-208 stable			Pb-207 stable

α-decay
Z: -2
N: -4

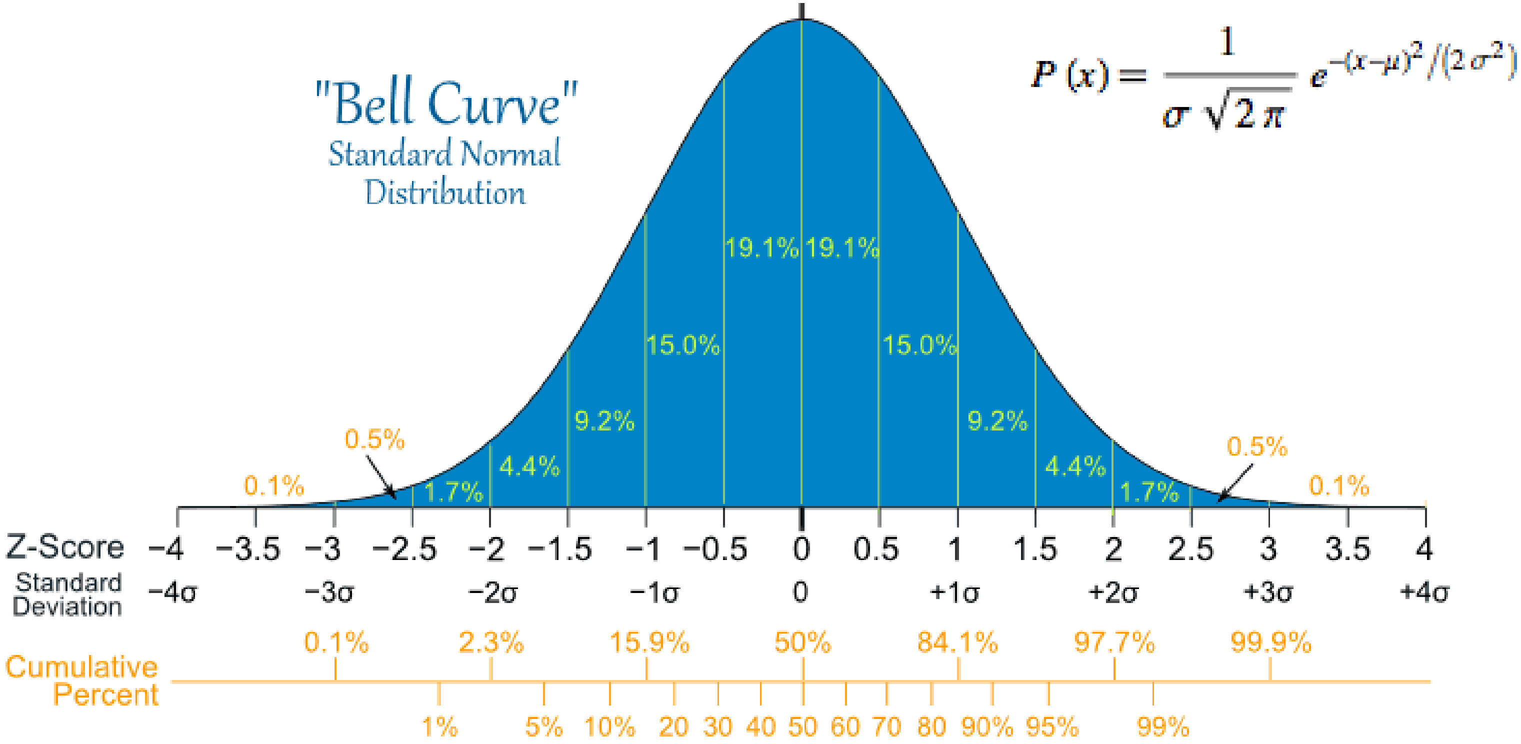
β-decay
Z: +1
N:+/-0

decay series of short-lived nuclides

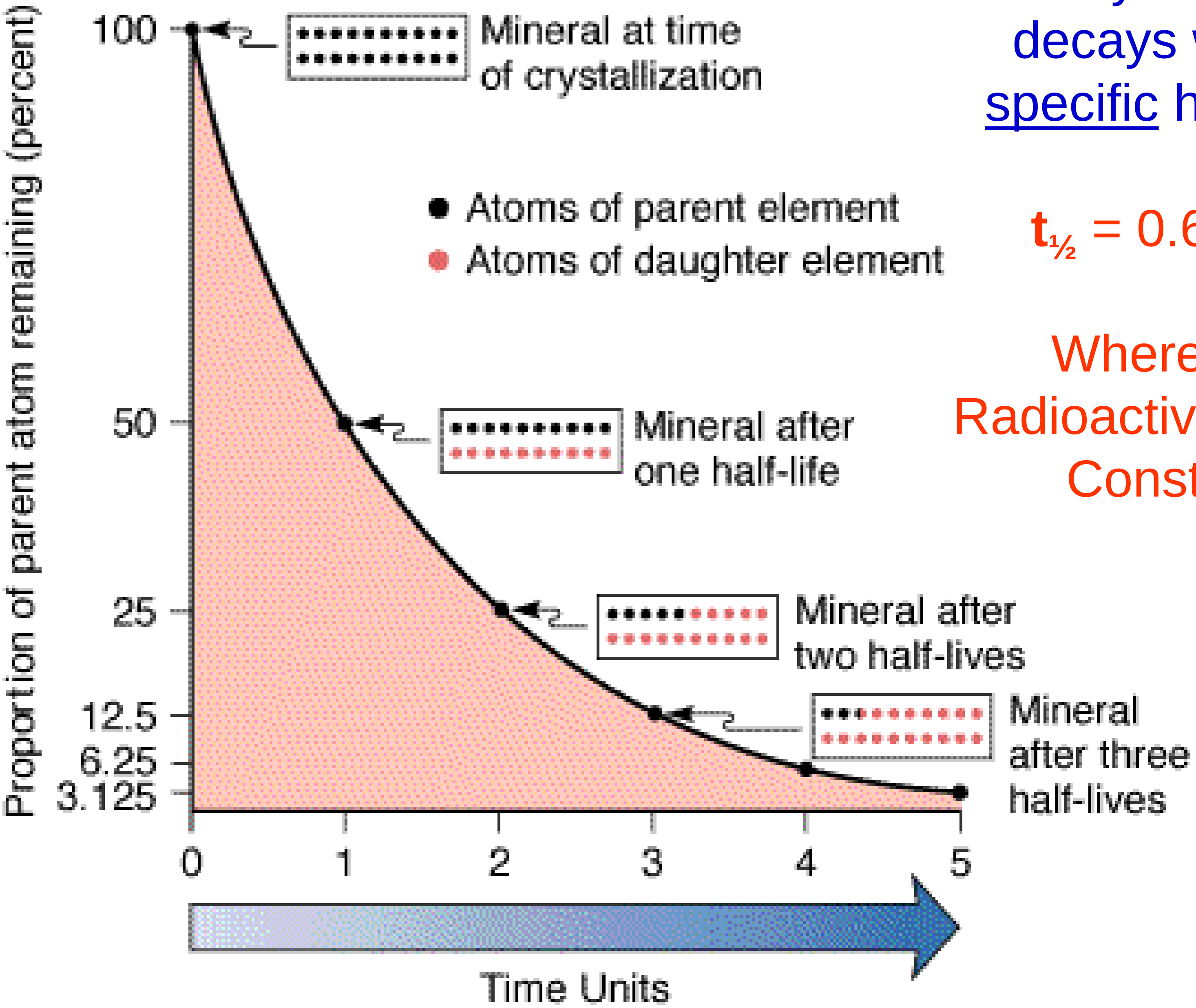
🟦 low
🟨 intermediate
🟧 high

Radioactive Decay: How does the process actually work?

Radioactive decay is a ***game of chance***. Once cannot pick out a single nucleus and predict how long it will be until it undergoes radioactive decay. However, each unstable nucleus has a specific probability of decaying in a given time interval. *In sufficient numbers*, the **probability** of decay becomes well defined.



https://www.mathsisfun.com/data/standard-normal-distribution.html



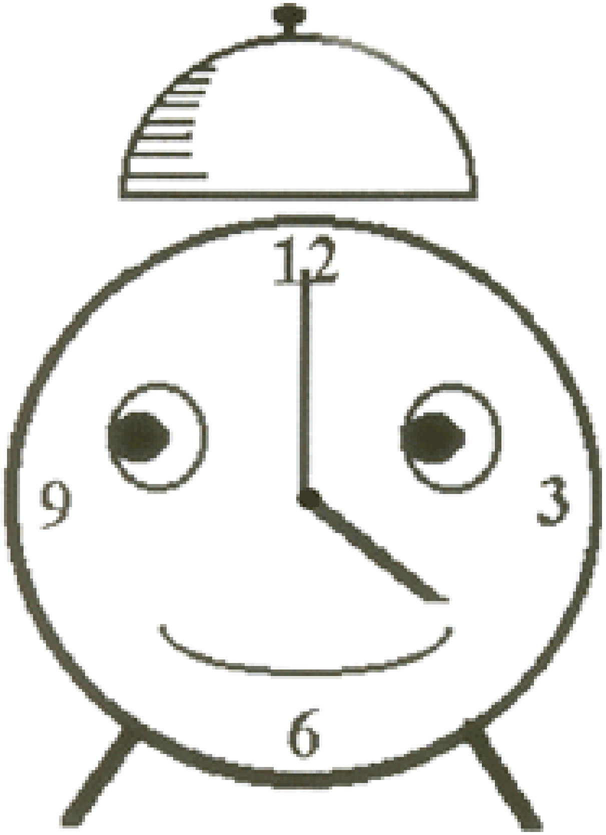
Every element decays with a specific half-life!

$t_{1/2} = 0.693/\lambda$

Where λ = Radioactive Decay Constant

What does this mean?

Radionuclides act as **CLOCKS** or **RATE TRACERS** on a variety of processes



The “classic” radioactive decay equation

Radioactive decay is a rate function.

$$\frac{dN}{dt} = -\lambda N$$ **= Activity**

N = number of atoms
λ = radioactive decay constant
N_0 = number of atoms at time = 0

$\lambda = 0.693/t_{½}$

$t_{½}$ = time it takes for half of the initial number of atoms to decay away.
T = 1/λ = mean life of a radionuclide

The basic equation for radioactive decay!!

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_o}^{N} \frac{dN}{N} = -\int_0^t \lambda dt$$

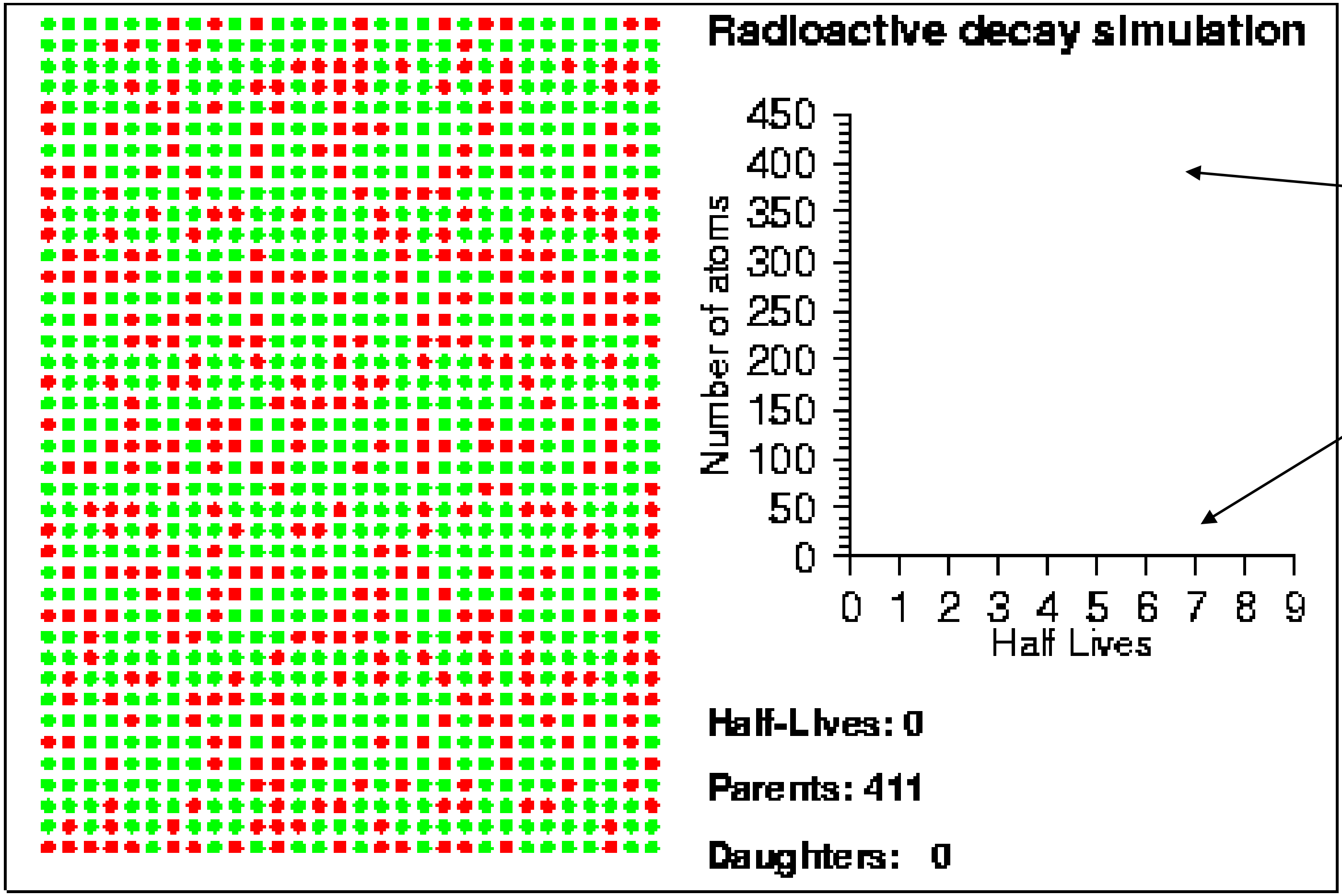
$$\ln N \Big|_{N_o}^{N} = -\lambda(t-0)$$

$$\ln(\frac{N}{N_o}) = -\lambda t$$

$$e^{\ln(\frac{N}{N_o})} = e^{-\lambda t}$$

$$\frac{N}{N_o} = e^{-\lambda t}$$

$$N = N_o e^{-\lambda t}$$



Half-Lives: 0

Parents: 411

Daughters: 0

$$D = N_0(1 - e^{-\lambda t})$$

$$N = N_0 e^{-\lambda t}$$

By comparing the number of parent and daughter atoms in a sample, we can estimate the amount of time since the sample was created. In the animation, the **radioactive isotopes** are represented by **red** circles, the **decay products** are the **blue** circles and the **neutral** isotopes are the **green** circles.

Case of a stable daughter (Geochronology in a nutshell)

P(arent) → D(aughter) Each parent atom which decays produces a stable daughter atom.

$$N_p^t = N_p^0 e^{-\lambda t}$$

Because each parent atom that is lost to decay produces a daughter atom, we should be able to determine the number of parent atoms at t = 0 by summing the number of parent atoms present today and the number of daughter atoms produced by decay of the parent since t = 0.

$$N_p^0 = N_p + N_d^*$$

(* refers to a radiogenic daughter, i.e. daughers produced by radioactive decay)

$$N_p^0 = \frac{N_p^t}{e^{-\lambda t}} = N_p e^{\lambda t} = N_p + N_d^*$$

$$N_d^* = N_p e^{\lambda t} - N_p = N_p(e^{\lambda t} - 1)$$

Calculate the time (t) elapsed since the composition of the sample was fixed solve the equation for t.

$$t = \frac{1}{\lambda} \ln \left[1 + \frac{N_d^*}{N_p}\right]$$

The total number of daughter atoms equals those present initially plus those produced by decay of the parent since the composition of the sample was fixed. **What if you have other daughter atoms???**

$$N_d = N_d^0 + N_d^*$$

$$N_d = N_d^0 + N_p(e^{\lambda t} - 1)$$

Measure N_d and N_p, BUT ***ESTIMATE*** N^0_d

In most cases: This is the big unknown!!

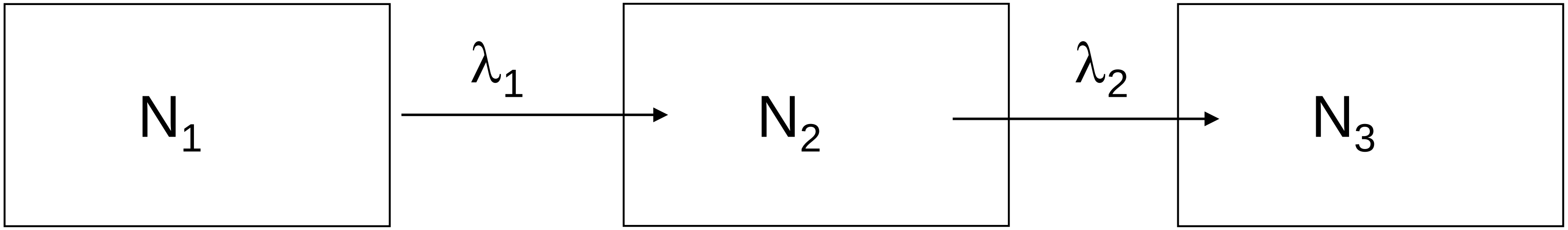
Let's get more complicated.

The Case of the Radioactive Daughter…….

A	→	B	→	C	→	………
N_1,λ_1		N_2,λ_2		N_3,λ_3		

Daughter B forms at the rate of parent A decay, but B also decays.
How do we find the activity of B at a particular time (The Bateman Equations)?

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$



$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0 [e^{-\lambda_1 t} - e^{-\lambda_2 t}] + N_2^0 e^{-\lambda_2 t}$$

Case of the Radioactive Daughter…….

4 cases:

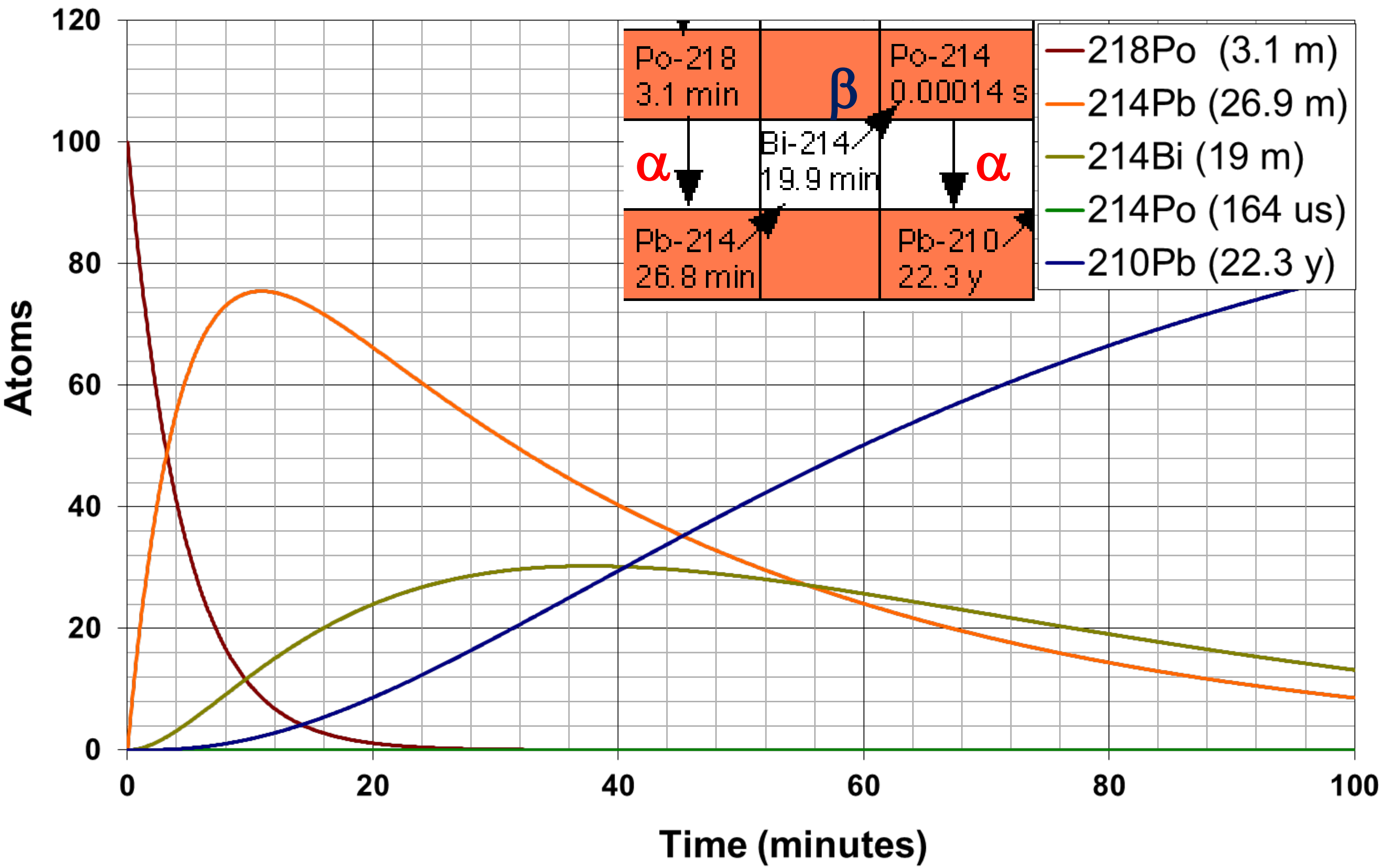
1. ${}^{N1}t_{½} < {}^{N2}t_{½}$. The half life of the parent is shorter than that of the daughter.

2. ${}^{N1}t_{½} \sim {}^{N2}t_{½}$. The half lives of the parent and daughter are similar.

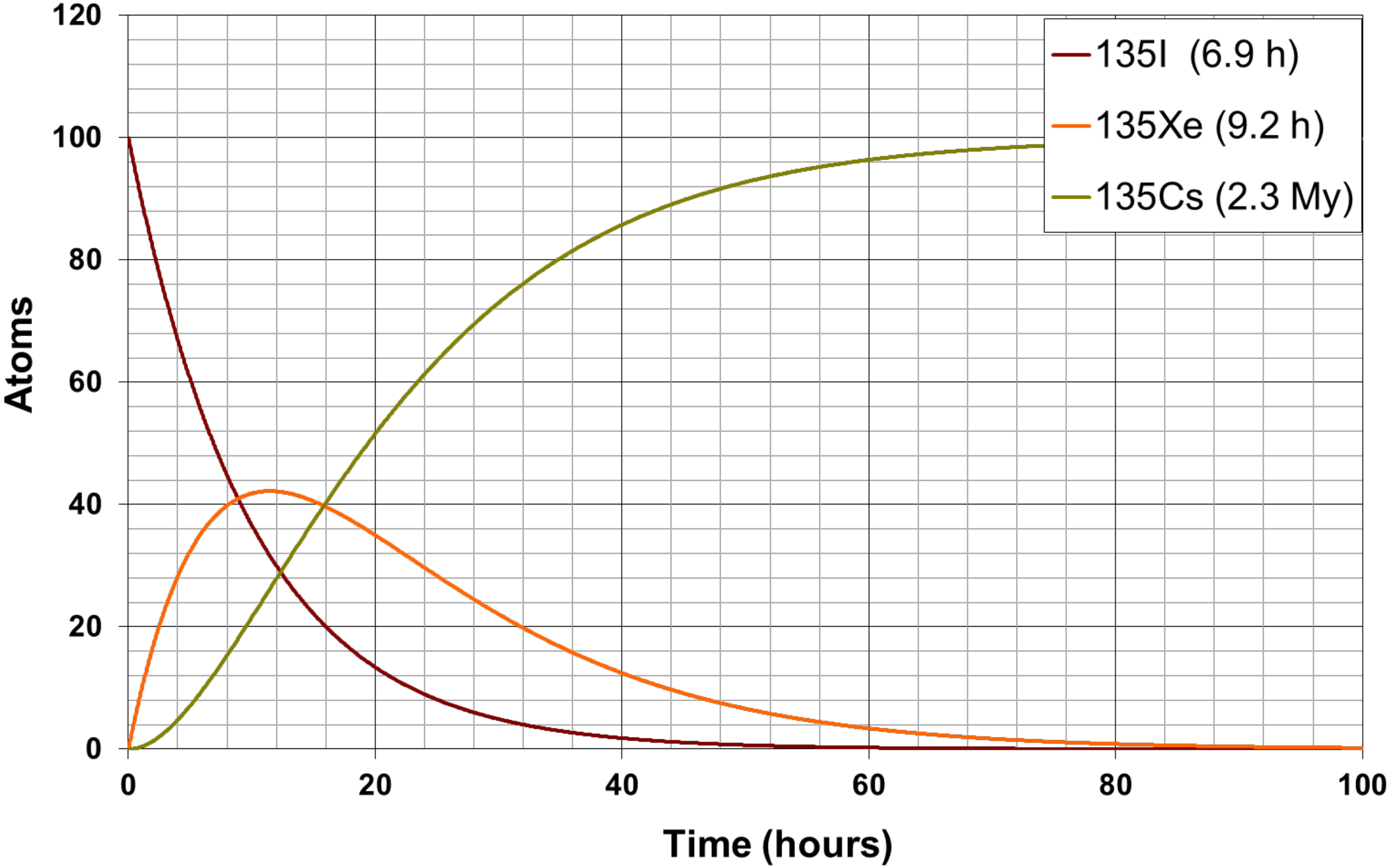
3. ${}^{N1}t_{½} > {}^{N2}t_{½}$. The half life of the parent is longer than that of the daughter.

4. ${}^{N1}t_{½} >> {}^{N2}t_{½}$. The half life of the parent is much longer than that of the daughter.

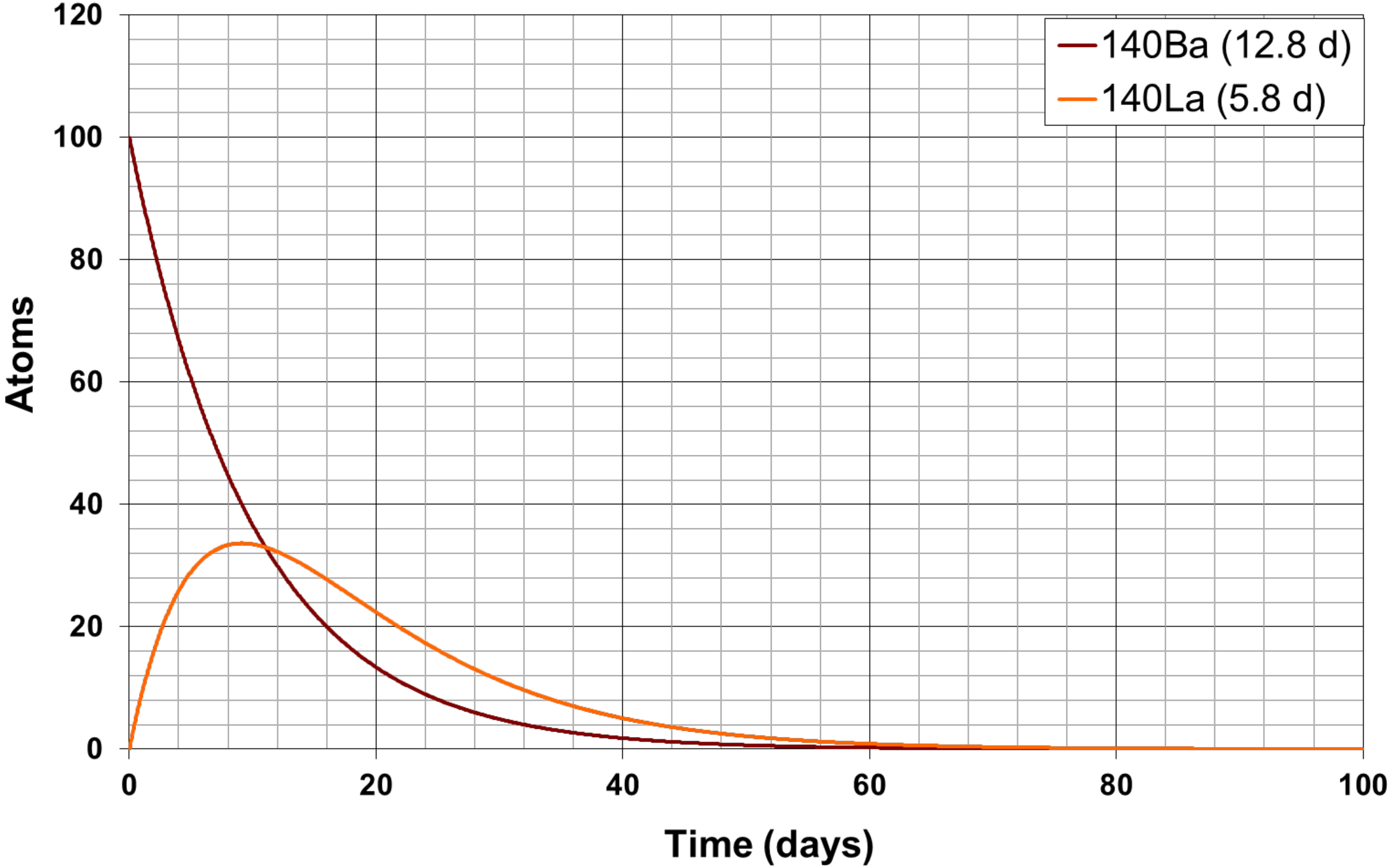
Case 1: $_{N1}t_{½} < {}_{N2}t_{½}$. The half life of the parent is shorter than that of the daughter.



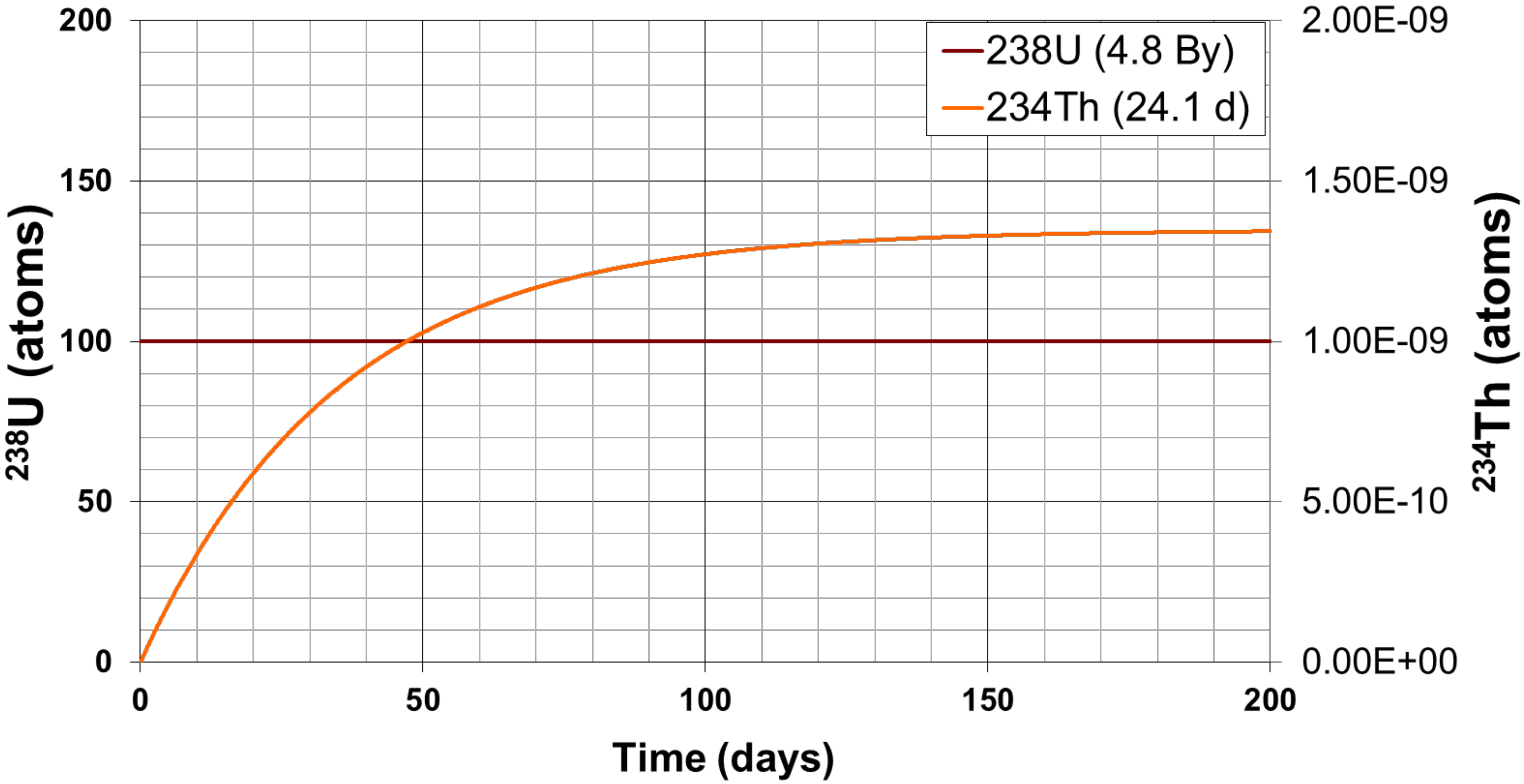
Case 2: ${}^{N1}t_{1/2} \sim {}^{N2}t_{1/2}$. The half lives of the parent and daughter are similar.



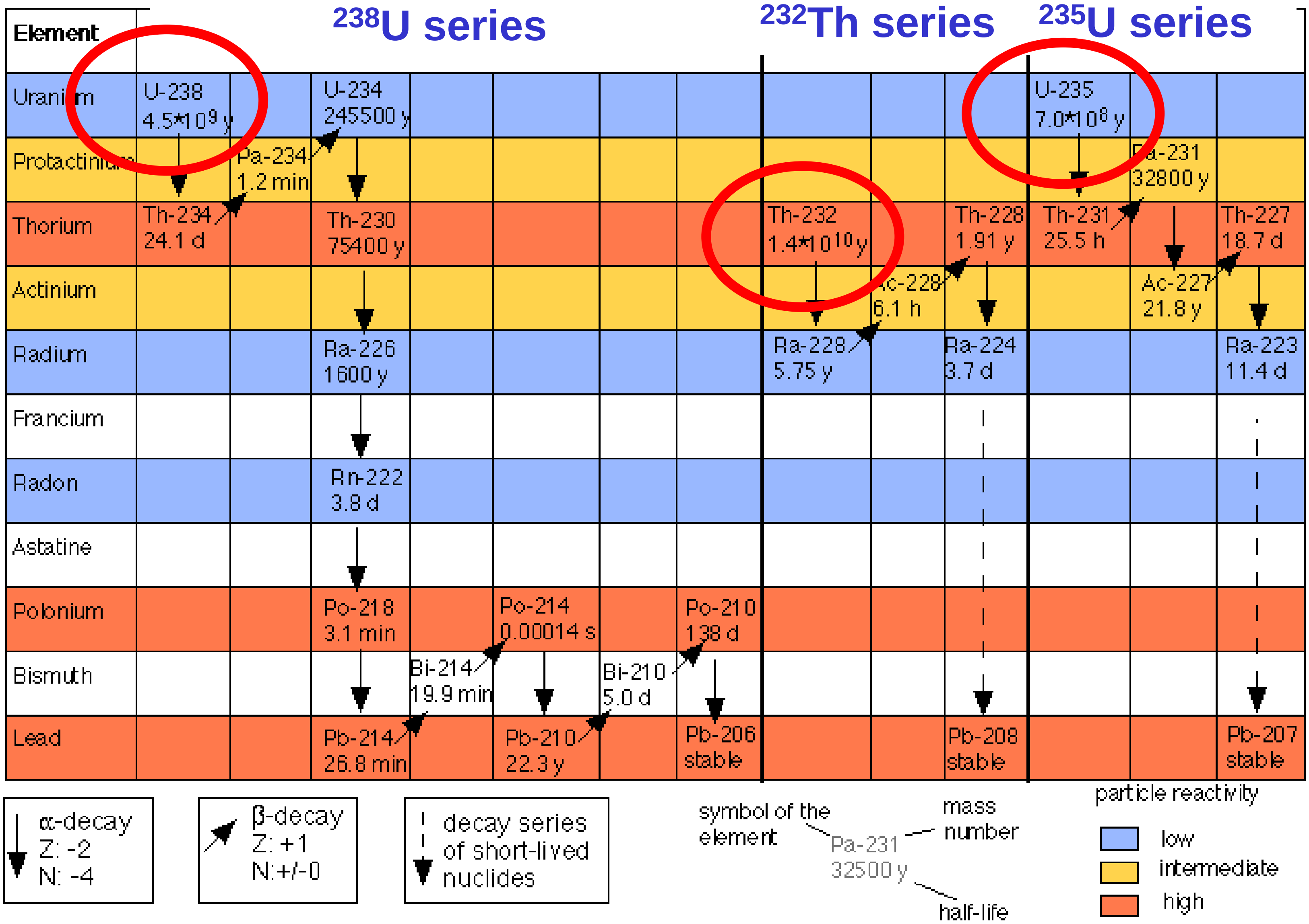
Case 3: $^{N1}t_{½} > {}^{N2}t_{½}$. The half life of the parent is longer than that of the daughter.



Case 4: ${}^{N1}t_{½}$ >> ${}^{N2}t_{½}$. The half life of the parent is much longer than that of the daughter.



For the naturally occurring radionuclides: **^{238}U, ^{235}U, and ^{232}Th** the half-lives of the parent nuclides are **much much longer** than their daughter products.



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Case 4: ${}^{N1}t_{½}$ >> ${}^{N2}t_{½}$. The half life of the parent is much longer than that of the daughter.

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \longrightarrow N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0 [e^{-\lambda_1 t} - e^{-\lambda_2 t}] + N_2^0 e^{-\lambda_2 t}$$

When $\lambda_1 << \lambda_2$ **=> $\lambda_1 N_1 = \lambda_2 N_2$**

$$A_1 = A_2$$

Secular Equilibrium

$$\lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = \lambda_4 N_4 = \ldots\ldots.$$

$$A_1 = A_2 = A_3 = A_4$$

Activity takes into account the radioactive decay component (and is what we actually measure)

Units of Radioactivity

- Curie (Ci)

  Originally based on disintegration rate of 1 g of radium, now defined as quantity of any radioactive nuclide in which the number of disintegrations per second (dps) is 3.7×10^{10}

- **Becquerel (Bq)**
  - **The SI unit, defined as 1 dps**
- **Disintegrations per minute (dpm)**
  - The favorite unit for oceanographers
- Sievert (Sv) = is dose and is the SI unit
  - Sv = equivalent dose, effective dose, and operational dose

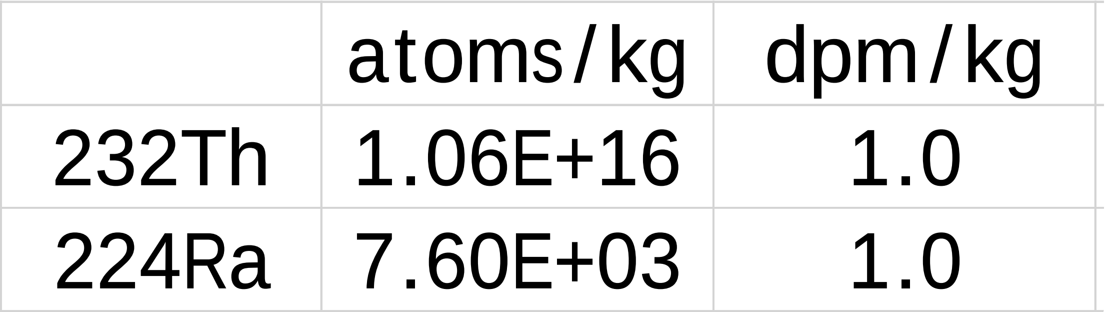
Example: Consider the ^{232}Th decay to ^{224}Ra

Here are the number of atoms typically found in marine systems

$$\text{Activity} = N\lambda \qquad \lambda = 0.693/t_{1/2}$$

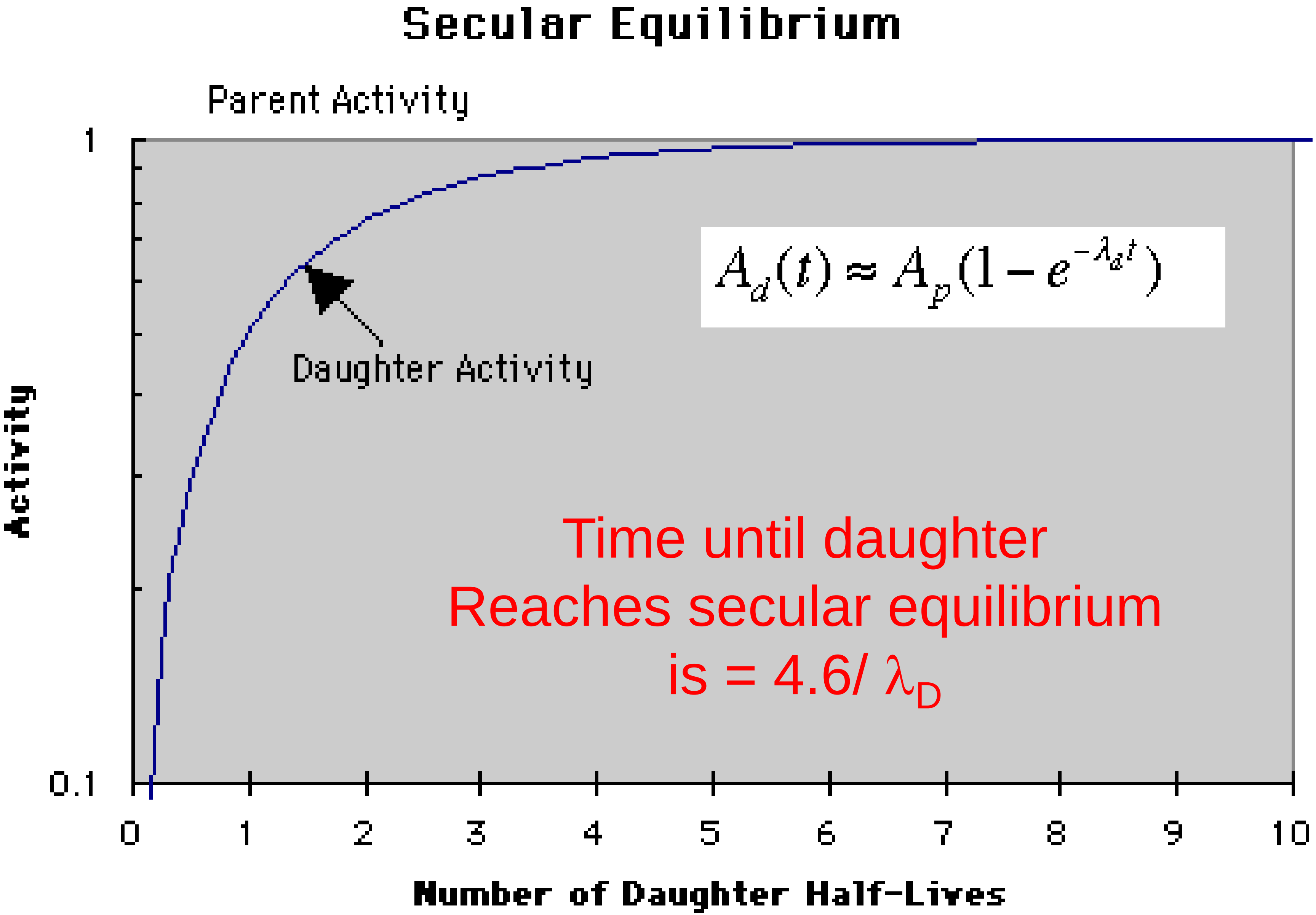
Where N = # of atoms, λ = decay constant, and $t_{1/2}$ = the half-life

^{224}Ra $t_{1/2}$ = 3.7 days ^{232}Th $t_{1/2}$ = 1.4 x 10^{10} years



	atoms/kg	dpm/kg
232Th	1.06E+16	1.0
224Ra	7.60E+03	1.0

The plot of parent and daughter activity in secular equilibrium is:



$$A_d(t) \approx A_p(1 - e^{-\lambda_d t})$$

Time until daughter
Reaches secular equilibrium
is = 4.6/ λ_D

Summary

1) Radioisotopes exist and decay using a variety of pathways that are known and measureable.

2) There are key equations used to describe the radioactive decay process and these equations can be vastly simplified under certain conditions

Any Questions?